

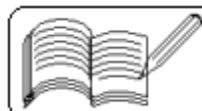
Instructions :

1. Answer all questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

SECTION - A

- Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question. 15

1. In ΔABC , if A is (1, -6), B is (-5, 2) and the centroid G is (-2, 1), then the coordinates of vertex C are
(a) (-2, 1) (b) (-2, 6) (c) (3, 2) (d) (-2, 7)
2. $d\{(a, 0), (0, b)\} = ?$
(a) a (b) b (c) $|a - b|$ (d) $\sqrt{a^2 + b^2}$
3. The t point of parabola $y^2 = 20x$ is ($t \in R$).
(a) $(5t, 4t^2)$ (b) $(5t^2, 4t)$ (c) $(5t^2, 10t)$ (d) $(t, 2t)$
4. If $y = 2x + c$ touches a parabola $y^2 = 16x$, then the value of c is
(a) 2 (b) -2 (c) 8 (d) $\sqrt{2}$
5. The equation of director circle of ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ is
(a) $x^2 + y^2 = 9$ (b) $x^2 + y^2 = 16$ (c) $x^2 + y^2 = 25$ (d) $x^2 + y^2 = 7$
6. The eccentricity of hyperbola $x^2 - y^2 = 144$ is
(a) $\sqrt{21}$ (b) $\sqrt{2}$ (c) $\sqrt{7}$ (d) $\sqrt{3}$
7. For distinct non - null vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d} \in R^3$, prove that $(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d})$ is
(a) $\begin{vmatrix} \bar{a} \cdot \bar{c} & \bar{a} \cdot \bar{d} \\ \bar{b} \cdot \bar{c} & \bar{b} \cdot \bar{d} \end{vmatrix}$ (b) $\begin{vmatrix} \bar{b} \cdot \bar{c} & \bar{b} \cdot \bar{d} \\ \bar{a} \cdot \bar{c} & \bar{a} \cdot \bar{d} \end{vmatrix}$ (c) $\begin{vmatrix} \bar{a} \cdot \bar{d} & \bar{a} \cdot \bar{c} \\ \bar{b} \cdot \bar{d} & \bar{b} \cdot \bar{c} \end{vmatrix}$ (d) $\begin{vmatrix} \bar{b} \cdot \bar{d} & \bar{b} \cdot \bar{c} \\ \bar{a} \cdot \bar{d} & \bar{a} \cdot \bar{c} \end{vmatrix}$
8. The projection of $\bar{a} = (1, 1, 1)$ on $\bar{b} = (2, 2, 1)$ is
(a) $\frac{5}{9}(2, 2, 1)$ (b) $(1, 3, 2)$ (c) $(0, 0, 1)$ (d) $\frac{1}{9}(1, 3, 2)$



9. The direction of a line passing through points (3, 2, 1) and (5, 6, 7) is
- (a) (8, 8, 8) (b) (2, 4, 3) (c) (4, 3, 2) (d) (2, 4, 6)
10. The perpendicular distance between $6x - 3y + 2z = 1$ and $12x - 6y + 4z = 21$ is
- (a) $\frac{63}{17}$ (b) $\frac{6}{31}$ (c) $\frac{12}{7}$ (d) $\frac{19}{14}$
11. The centre of sphere $|\vec{r}|^2 - \vec{r} \cdot (2, 4, 6) + 5 = 0$ is
- (a) (2, 4, 6) (b) (1, 2, 3) (c) (2, 1, 3) (d) (2, 3, 5)
12. N (a, d) form of the set $\{x / |x + 1| < 3, x \in \mathbb{R}\}$ is
- (a) N(1, 3) (b) N(2, 3) (c) N(3, 1) (d) N(-1, 3)
13. For $\sqrt{x} - \sqrt{y} = \sqrt{a}$, $a > 0$, $\frac{dy}{dx} = ?$
- (a) \sqrt{x} (b) \sqrt{y} (c) $\sqrt{\frac{y}{x}}$ (d) $\sqrt{\frac{x}{y}}$
14. $\int \frac{dx}{x^2 + 4x + 5} = ?$
- (a) $\tan^{-1}(x+5) + c$ (b) $\tan^{-1}(x+4) + x$ (c) $\tan^{-1}(x+2) + c$ (d) $\tan^{-1}(5x+4) + c$
15. $\int_1^4 \left(\frac{x^2 + 1}{x} \right)^{-1} dx = ?$
- (a) $\log \left| \frac{17}{2} \right|$ (b) $\frac{1}{2} \log \left| \frac{17}{2} \right|$ (c) $2 \log |17|$ (d) none of these

SECTION B

- Answer the following 16 to 30 questions. Each question carries one mark.

15

16. If a line $(a + 3)x + (a^2 - 9)y + (a - 3) = 0$ passes through origin, then find the value of a.

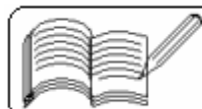
OR

Find k, if the following lines are concurrent.

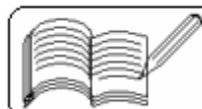
$$2x + 5y + 3 = 0$$

$$5x - 9y + k = 0 \text{ and}$$

$$x - 2y + 1 = 0.$$



17. Find the equation of a parabola whose focus is $S(4, 0)$ and the equation of its directrix is $x + 4 = 0$.
18. Find the tangents to the parabola $y^2 = 8x$ that is perpendicular to the line $x + 2y + 5 = 0$.
19. Prove that $(\bar{x} - \bar{y}) \times (\bar{x} + \bar{y}) = 2(\bar{x} \times \bar{y})$.
20. Obtain the cosine formula for a triangle by using vectors.
21. If the equation $|\bar{r}|^2 - \bar{r} \cdot (2, 1, 1) + 3 = 0$ represents a sphere, then find its radius.
22. Obtain the equation of a sphere if the extremities of its diameter are $(1, 1, 1)$ and $(2, 2, 1)$.
23. Find k if $f(x) = \begin{cases} kx - 1, & x < 2 \\ x, & x \geq 2 \end{cases}$ is continuous at $x = 2$. OR
- Obtain $\lim_{x \rightarrow 0} \frac{(2006)^x + (2005)^x - 2}{x}$.
24. Prove that $f(x) = e^{\frac{1}{x}}$ is a decreasing function for $x > 0$.
25. Find the approximate value of $\sqrt{28}$.
26. Verify Rolle's theorem for $f(x) = x^2$, $x \in [-2, 2]$.
27. Evaluate : $\int \frac{\log x}{x} dx$ OR Evaluate : $\int [\sin^2 x + \sin 2x] e^x dx$.
28. Show that $\int_x^\pi x f(\sin x) dx = \frac{\pi}{2} \int_x^\pi f(\sin x) dx$.
29. Solve the differential equation $x \frac{dy}{dx} = y + 2$.



30. Write down the order of the differential equation $\frac{d^2y}{dx^2} + 3y = 0$.

SECTION C

- Answer the following 31 to 40 questions as directed. Each question carries two marks. 20

31. Let A be (3, -1) and B (0, 4). If $P(x, y) \in \overline{AB}$, obtain the maximum and minimum values of $3y - x$.

OR

Find the equations of lines containing the diagonals of the rectangle formed by the lines $x = 2$, $x = -1$, $y = 6$ and $y = -2$.

32. Find the maximum and minimum distances of points on the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ from the point (10, 7).

OR

Prove that for every value of k, the circle $2x^2 + 2y^2 - 12x + ky + 18 = 0$ touches the X-axis.

33. Find the equation of the ellipse passing through the points (1, 4) and (-6, 1).

34. Find the measure of the angle between the asymptotes of hyperbola $3x^2 - 2y^2 = 1$.

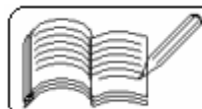
35. Find a unit vector orthogonal to (2, 1, 1) and (1, 2, 3).

36. Find the area of a parallelogram if its diagonals are $2\bar{i} + \bar{k}$ and $\bar{i} + \bar{j} + \bar{k}$.

37. Obtain : $\lim_{x \rightarrow \pi} \frac{\sqrt{10 + \cos x} - 3}{(\pi - x)^2}$ OR Obtain $\lim_{x \rightarrow 1} (1 - x) \tan\left(\frac{\pi x}{2}\right)$

38. Find : $\lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{1}{4r^2 - 1} \right)$.

39. Find : $\int \frac{\sin 2x \, dx}{m^2 \sin^2 x - n^2 \cos^2 x}$



40. Evaluate : $\int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} dx$ OR Show that : $\int_0^{\frac{\pi}{2}} \frac{dx}{2 + \cos x} = \frac{\pi}{3\sqrt{3}}$.

SECTION D

- Answer the following 41 to 50 questions as directed. Each question carries 3 marks. 30

41. If G and I are respectively the centroid and incentre of the triangle whose vertices are A (-2, -1), B (1, -1) and C (1, 3), Find IG.

42. If circle $x^2 + y^2 + 2x + fy + k = 0$ touches both the axes, then find f and k.

43. If $\bar{x} + \bar{y} + \bar{z} = \bar{0}$, then prove that $\bar{x} \times \bar{y} = \bar{y} \times \bar{z} = \bar{z} \times \bar{x}$ OR

If the vectors (a, 1, 1), (1, b, 1) and (1, 1, c) are coplanar vectors, then show that $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$.

44. Find the shortest distance between the lines $x = y = z$ and $\frac{x+1}{1} = \frac{y}{2} = \frac{z}{3}$.

45. Find the vector and Cartesian equations of plane and distance from origin to the plane which passes through points A (1, 1, 0), B (0, 1, 1) and C (1, 0, 1).

46. Obtain : $\lim_{x \rightarrow 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$, m, n \in N.

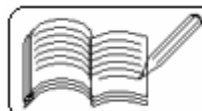
47. If $y = a \cos(\log x) + b \sin(\log x)$, then prove that $x^2 y_2 + xy_1 + y = 0$.

48. Using the mean value theorem, prove that

$$\frac{1}{1+x^2} < \frac{\tan^{-1}x - \tan^{-1}y}{x-y} < \frac{1}{1+y^2}, \quad (x > y > 0) \quad \text{OR}$$

Show that the curves $y = ax^3$ and $x^2 + 3y^2 = b^2$ are orthogonal curves. (a > 0, b > 0).

49. Solve the differential equation : $x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$.



50. If time taken for horizontal range R is T , prove that the angle of projection has measure $\tan^{-1} \left(\frac{gT^2}{2R} \right)$.

OR

The velocity of a projectile at the maximum height is $\sqrt{\frac{2}{5}}$ times its velocity at half the maximum height. Prove that angle of projection has measure $\frac{\pi}{3}$.

SECTION E

- Answer the following 51 to 54 questions. Each question carries 5 marks. 20

51. In $\triangle ABC$, C is $(4, -1)$. The line containing the altitude from A is $3x + y + 11 = 0$ and the line containing the median \overline{AD} through A is $x + 2y + 7 = 0$. Find the equations of lines containing the three sides of the triangle.

OR

Find the equation of the line that passes through the point of intersection of $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ and that cuts off intercepts of equal magnitudes on the two axes.

52. $f(x) = \begin{cases} e^x, & x \geq 0 \\ \log(x + e), & x < 0 \end{cases}$

Is f continuous at $x = 0$? Is it differentiable at $x = 0$? Why?

53. Obtain : $\int \frac{dx}{\sin x + \sec x}$.

54. Obtain : $\int_1^4 x^3 dx$ as the limit of a sum.

OR

Prove that $\int_1^{\frac{\pi}{2}} \frac{x \sec x}{1 + \tan x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$.

