Instructions :

1. Answer all questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

SECTION - A

- Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question. **15**

1. In Δ ABC, if A is (1, - 6), B is (- 5, 2) and the centroid G is (- 2, 1), then the coordinates of vertex C are
   (a) (- 2, 1)   (b) (- 2, 6)   (c) (3, 2)   (d) (- 2, 7)

2. $d\{(a, 0), (0, b)\} = ?$
   (a) a   (b) b   (c) $|a - b|$ (d) $\sqrt{a^2 + b^2}$

3. The t point of parabola $y^2 = 20x$ is ..... ($t \in R$).
   (a) $(5t, 4t^2)$ (b) $(5t^2, 4t)$ (c) $(5t^2, 10t)$ (d) $(t, 2t)$

4. If $y = 2x + c$ touches a parabola $y^2 = 16x$, then the value of c is .....
   (a) 2   (b) - 2   (c) 8   (d) $\sqrt{2}$

5. The equation of director circle of ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ is .....
   (a) $x^2 + y^2 = 9$ (b) $x^2 + y^2 = 16$ (c) $x^2 + y^2 = 25$ (d) $x^2 + y^2 = 7$

6. The eccentricity of hyperbola $x^2 - y^2 = 144$ is .....
   (a) $\sqrt{21}$ (b) $\sqrt{2}$ (c) $\sqrt{7}$ (d) $\sqrt{3}$

7. For distinct non - null vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d} \in R^3$, prove that $(\bar{a} \times \bar{b}) . (\bar{c} \times \bar{d})$ is .....
   (a) $\begin{vmatrix} \bar{a}.\bar{c} & \bar{a}.\bar{d} \\ \bar{b}.\bar{c} & \bar{b}.\bar{d} \end{vmatrix}$ (b) $\begin{vmatrix} \bar{b}.\bar{c} & \bar{b}.\bar{d} \\ \bar{a}.\bar{c} & \bar{a}.\bar{d} \end{vmatrix}$ (c) $\begin{vmatrix} \bar{a}.\bar{d} & \bar{a}.\bar{c} \\ \bar{b}.\bar{d} & \bar{b}.\bar{c} \end{vmatrix}$ (d) $\begin{vmatrix} \bar{b}.\bar{d} & \bar{b}.\bar{c} \\ \bar{a}.\bar{d} & \bar{a}.\bar{c} \end{vmatrix}$

8. The projection of $\bar{a} = (1, 1, 1)$ on $\bar{b} = (2, 2, 1)$ is .....
   (a) $\frac{5}{9}(2, 2, 1)$ (b) (1, 3, 2)   (c) (0, 0, 1)   (d) $\frac{1}{9}(1, 3, 2)$

9. The direction of a line passing through points (3, 2, 1) and (5, 6, 7) is .....
(a) (8, 8, 8) (b) (2, 4, 3) (c) (4, 3, 2) (d) (2, 4, 6)

10. The perpendicular distance between $6x - 3y + 2z = 1$ and $12x - 6y + 4z = 21$ is .....
(a) $\frac{63}{17}$ (b) $\frac{6}{31}$ (c) $\frac{12}{7}$ (d) $\frac{19}{14}$

11. The centre of sphere $|\bar{r}|^2 - \bar{r}.(2, 4, 6) + 5 = 0$ is .....
(a) (2, 4, 6) (b) (1, 2, 3) (c) (2, 1, 3) (d) (2, 3, 5)

12. N (a, d) form of the set $\{ x \, / \, |x + 1| < 3, \ x \in R \}$ is .....
(a) N (1, 3) (b) N (2, 3) (c) N (3, 1) (d) N (- 1, 3)

13. For $\sqrt{x} \cdot \sqrt{y} = \sqrt{a}$, $a > 0$, $\frac{dy}{dx} = ?$
(a) $\sqrt{x}$ (b) $\sqrt{y}$ (c) $\sqrt{\frac{y}{x}}$ (d) $\sqrt{\frac{x}{y}}$

14. $\int \frac{dx}{x^2 + 4x + 5} = ?$
(a) $\tan^{-1}(x + 5) + c$ (b) $\tan^{-1}(x + 4) + x$ (c) $\tan^{-1}(x + 2) + c$ (d) $\tan^{-1}(5x + 4) + c$

15. $\int_1^4 \left(\frac{x^2 + 1}{x} \right)^{-1} dx = ?$
(a) $\log \left| \frac{17}{2} \right|$ (b) $\frac{1}{2} \log \left| \frac{17}{2} \right|$ (c) $2 \log |17|$ (d) none of these

SECTION B

- **Answer the following 16 to 30 questions. Each question carries one mark.** **15**

16. If a line $(a + 3)x + (a^2 - 9)y + (a - 3) = 0$ passes through origin, then find the value of a.

OR

Find k, if the following lines are concurrent.
$2x + 5y + 3 = 0$
$5x - 9y + k = 0$ and
$x - 2y + 1 = 0.$

17. Find the equation of a parabola whose focus is S (4, 0) and the equation of its directrix is $x + 4 = 0$.

18. Find the tangents to the parabola $y^2 = 8x$ that is perpendicular to the line $x + 2y + 5 = 0$.

19. Prove that $(\bar{x} - \bar{y}) \times (\bar{x} + \bar{y}) = 2(\bar{x} \times \bar{y})$.

20. Obtain the cosine formula for a triangle by using vectors.

21. If the equation $|\bar{r}|^2 - \bar{r} \cdot (2, 1, 1) + 3 = 0$ represents a sphere, then find its radius.

22. Obtain the equation of a sphere if the extremities of its diameter are (1, 1, 1) and (2, 2, 1).

23. Find k if $f(x) = \begin{cases} kx - 1, & x < 2 \\ x, & x \geq 2 \end{cases}$ is continuous at $x = 2$. **OR**

Obtain $\lim_{x \to 0} \dfrac{(2006)^x + (2005)^x - 2}{x}$.

24. Prove that $f(x) = e^{\frac{1}{x}}$ is a decreasing function for $x > 0$.

25. Find the approximate value of $\sqrt{28}$.

26. Verify Rolle's theorem for $f(x) = x^2,\ x \in [-2, 2]$.

27. Evaluate : $\int \dfrac{\log x}{x}\, dx$ **OR** Evaluate : $\int [\sin^2 x + \sin 2x]\, e^x\, dx$.

28. Show that $\int_x^{\pi} x\, f(\sin x)\, dx = \dfrac{\pi}{2} \int_x^{\pi} f(\sin x)\, dx$.

29. Solve the differential equation $x \dfrac{dy}{dx} = y + 2$.

30. Write down the order of the differential equation $\frac{d^2y}{dx^2} + 3y = 0$.

SECTION C

- **Answer the following 31 to 40 questions as directed. Each question carries two marks.** **20**

31. Let A be $(3, -1)$ and B $(0, 4)$. If $P(x, y) \in \overline{AB}$, obtain the maximum and minimum values of $3y - x$.

OR

Find the equations of lines containing the diagonals of the rectangle formed by the lines $x = 2$, $x = -1$, $y = 6$ and $y = -2$.

32. Find the maximum and minimum distances of points on the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ from the point $(10, 7)$.

OR

Prove that for every value of k, the circle $2x^2 + 2y^2 - 12x + ky + 18 = 0$ touches the X - axis.

33. Find the equation of the ellipse passing through the points $(1, 4)$ and $(-6, 1)$.

34. Find the measure of the angle between the asymptotes of hyperbola $3x^2 - 2y^2 = 1$.

35. Find a unit vector orthogonal to $(2, 1, 1)$ and $(1, 2, 3)$.

36. Find the area of a parallelogram if its diagonals are $2\bar{i} + \bar{k}$ and $\bar{i} + \bar{j} + \bar{k}$.

37. Obtain : $\lim_{x \to \pi} \frac{\sqrt{10 + \cos x} - 3}{(\pi - x)^2}$ **OR** Obtain $\lim_{x \to 1} (1 - x) \tan\left(\frac{\pi x}{2}\right)$

38. Find : $\lim_{n \to \infty} \sum_{r=1}^{n} \left(\frac{1}{4r^2 - 1}\right)$.

39. Find : $\int \frac{\sin 2x \, dx}{m^2 \sin^2 x - n^2 \cos^2 x}$

40. Evaluate : $\int_0^1 x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$ **OR** Show that : $\int_0^{\frac{\pi}{2}} \frac{dx}{2+\cos x} = \frac{\pi}{3\sqrt{3}}$.

SECTION D

- **Answer the following 41 to 50 questions as directed. Each question carries 3 marks.** **30**

41. If G and I are respectively the centroid and incentre of the triangle whose vertices are A (− 2, − 1), B (1, − 1) and C (1, 3), Find IG.

42. If circle $x^2 + y^2 + 2x + fy + k = 0$ touches both the axes, then find f and k.

43. If $\bar{x} + \bar{y} + \bar{z} = \bar{0}$, then prove that $\bar{x} \times \bar{y} = \bar{y} \times \bar{z} = \bar{z} \times \bar{x}$ **OR**

If the vectors (a, 1, 1), (1, b, 1) and (1, 1, c) are coplanar vectors, then show that $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$.

44. Find the shortest distance between the lines $x = y = z$ and $\frac{x+1}{1} = \frac{y}{2} = \frac{z}{3}$.

45. Find the vector and Cartesian equations of plane and distance from origin to the plane which passes through points A (1, 1, 0), B (0, 1, 1) and C (1, 0, 1).

46. Obtain : $\lim_{x \to 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$, $m, n \in N$.

47. If $y = a\cos(\log x) + b\sin(\log x)$, then prove that $x^2 y_2 + x y_1 + y = 0$.

48. Using the mean value theorem, prove that

$\frac{1}{1+x^2} < \frac{\tan^{-1}x - \tan^{-1}y}{x-y} < \frac{1}{1+y^2}$, $(x > y > 0)$ **OR**

Show that the curves $y = ax^3$ and $x^2 + 3y^2 = b^2$ are orthogonal curves. $(a > 0, b > 0)$.

49. Solve the differential equation : $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$.

50. If time taken for horizontal range R is T, prove that the angle of projection has measure $\tan^{-1}\left(\frac{gT^2}{2R}\right)$.

OR

The velocity of a projectile at the maximum height is $\sqrt{\frac{2}{5}}$ times its velocity at half the maximum height. Prove that angle of projection has measure $\frac{\pi}{3}$.

SECTION E

- **Answer the following 51 to 54 questions. Each question carries 5 marks. 20**

51. In Δ ABC, C is $(4, -1)$. The line containing the altitude from A is $3x + y + 11 = 0$ and the line containing the median $\overline{AD}$ through A is $x + 2y + 7 = 0$. Find the equations of lines containing the three sides of the triangle.

OR

Find the equation of the line that passes through the point of intersection of $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ and that cuts off intercepts of equal magnitudes on the two axes.

52. $f(x) = \begin{cases} e^x, & x \geq 0 \\ \log(x + e), & x < 0 \end{cases}$

Is f continuous at $x = 0$? Is it differentiable at $x = 0$? Why ?

53. Obtain : $\int \frac{dx}{\sin x + \sec x}$.

54. Obtain : $\int_1^4 x^3\, dx$ as the limit of a sum.

OR

Prove that $\int_1^{\frac{\pi}{2}} \frac{x \sec x}{1 + \tan x}\, dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$.