Instructions :

1. **Answer all questions.**
2. **Write your answers according to the instructions given below with the questions.**
3. **Begin each section on a new page.**

SECTION - A

- **Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question.** **15**

1. **Find the value of a, if P (2, 3) is circumcentre of the triangle with vertices A (a, 6), B (5, 1) and C (4, 6).**
   (a) - 4 (b) 1 (c) 4 (d) 0

2. **Find α if a line $x + y + 1 = 0$ is converted in the form of a line $x\cos\alpha + y\sin\alpha = p$.**
   (a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{4}$ (d) $\frac{7\pi}{4}$

3. **If the circle $x^2 + y^2 + 4x + Ky - 4 = 0$ touches both the axes, then find K.**
   (a) ± 8 (b) ± 4 (c) ± 2 (d) ± 1

4. **Obtain the equation of a parabola having focus (0, - 2) and the equation of directrix y = 2. The vertex of the parabola is (0, 0).**
   (a) $x^2 = -8y$ (b) $y^2 = 8x$ (c) $x^2 = 8y$ (d) $y^2 = -8x$

5. **Find the radius of a director - circle of an ellipse $4x^2 + 9y^2 = 36$.**
   (a) $\sqrt{5}$ (b) $\sqrt{13}$ (c) $\sqrt{10}$ (d) 5

6. **If $|\bar{a}| = 10$, $|\bar{b}| = 2$ and $\bar{a}.\bar{b} = 12$, then find $|\bar{a} \times \bar{b}|$**
   (a) 12 (b) 14 (c) 16 (d) 18

7. **Find magnitude of projection of vector $\bar{i} + \bar{j} + \bar{k}$ on $\bar{j}$.**
   (a) - 1 (b) 0 (c) 1 (d) 2

8. **Find the measure of the angle between planes $\bar{r} . (1, 2, 1) = 1$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$.**
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) none of them

9. Find $\lim_{x \to 0} \frac{(1 + x)^{\frac{1}{3}} - 1}{x}$

(a) 0 (b) 1 (c) $\frac{1}{3}$ (d) none of these

10. Find $\frac{d}{dx}\left[\tan^{-1}\left(\frac{1 - \cos x}{1 + \cos x} \right)^{\frac{1}{2}} \right] : \pi < x < 2\pi.$

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

11. Find c applying Rolle's theorem to $f(x) = 1 + \sin x, x \in [0, \pi]$.

(a) 0 (b) $\frac{\pi}{4}$ (c) π (d) $\frac{\pi}{2}$

12. Evaluate : $\int_1^{\sqrt{3}} \frac{1}{1 + x^2} dx$.

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

13. Find the area of the region bounded by the curve $y = \tan x$, X - axis and the lines $x = 0$ and $x = \frac{\pi}{4}$.

(a) log 2 (b) $\frac{3}{2}$ log 2 (c) $\frac{1}{2}$ log 2 (d) 2 log 2

14. Determine the degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx} \right)^2 = x^2 \log\left(\frac{d^2y}{dx^2} \right)$.

(a) 1 (b) 2 (c) 0 (d) not defined

15. A stone falls from a tower of height 40 m. What will be its velocity when it reaches the ground level.

(a) 14 m/s (b) 28 m/s (c) 21 m/s (d) 7 m/s

SECTION B

- **Answer the following 16 to 30 questions. Each question carries one mark.** **15**

16. Find the point A on the X - axis which is at the distance of 5 units from point B (2, - 3).

17. Obtain the equation of a circle which touches the X - axis, given that the equations of lines containing two of the diameters of the circle are $3x - 2y - 5 = 0$ and $x + y = 5$.

18. Find the focus of a parabola $y^2 + 6y - 2x + 5 = 0$.

19. The equations of the asymptotes of hyperbola are $3x + 4y = 2$ and $4x - 3y = 2$. Find the eccentricity.

20. Find the unit vector in the direction of vector (1, 2, 3).

21. Find the area of a parallelogram, if the diagonals are $2\bar{i} + \bar{k}$ and $\bar{i} + \bar{j} + \bar{k}$.

22. Represent the equation of the line $\frac{3 - x}{1} = \frac{2 - y}{3} = \frac{1 - z}{4}$ in the vector form.

23. Find the length of a chord cut by sphere $x^2 + y^2 + z^2 - x - y - z = 0$ on any axis.

24. If $f'(x) = f(x)$ and $f(0) = 1$, then find out the value of $\lim_{x \to 0} \frac{f(x) - 1}{x}$.

25. Evaluate : $\int x^{4x} (1 + \log x)\, dx,\ x > 0$.

26. Evaluate : $\int \left(\frac{1 + x}{x^2} \right) e^{-x}\, dx$.

27. If $\int_1^k f(x)\, dx = 47$; $f(x) = \begin{cases} 2x + 8, & \text{if } 1 \le x \le 2 \\ 6x, & \text{if } 2 \le x \le k \end{cases}$ then find k.

28. Find the length of subtangent of $y = e^{\frac{x}{c}}$.

29. If a distance of 150 cm. is traveled in 30 seconds with an initial velocity of 10 cm/s, find the constant acceleration (retardation).

30. If the maximum horizontal range is 200 m, find the minimum velocity for that.

SECTION C

- **Answer the following 31 to 40 questions as directed. Each question carries two marks. 20**

31. A line passing through (2, 4) intersects the X - axis and Y - axis at A and B respectively. Find the equation of the locus of the mid - point of $\overline{AB}$.

32. For the parabola $x^2 = 12y$, find the area of the triangle, whose vertices are the vertex of the parabola and the two end - points of its latus rectum.

33. Find the equation of the ellipse which is passing through the points (1, 4) and (- 6, 1).

34. Find the equation of hyperbola for which the distance from one vertex to two foci are 9 and 1.

OR

Find the measure of angle between the asymptotes of hyperbola $3x^2 - 2y^2 = 1$.

35. If $\bar{x} \cdot \bar{y} = \bar{x} \cdot \bar{z}$, $\bar{x} \times \bar{y} = \bar{x} \times \bar{z}$ and $\bar{x} \neq \bar{0}$, then prove that $\bar{y} = \bar{z}$.

36. If $\bar{a} \cdot \bar{b} = \bar{a} \cdot \bar{c} = 0$, $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$, then prove that $\bar{a} = \pm 2(\bar{b} \times \bar{c})$, where $(\bar{b} \wedge \bar{c}) = \frac{\pi}{6}$.

37. Find the equation of a sphere given that its centre is (1, 1, 0) and that it touches the plane $2x + 2y + z + 5 = 0$.

38. If $y = \tan^{-1}\left(\frac{5x}{1 - 6x^2}\right)$, then find $\frac{dy}{dx}$. **OR**

$f(x) = [x]$. Is f continuous and differentiable at $x = 1$?

39. Find the measure of the angle between the curves $y = \sin x$ and $y = \cos x$, $0 < x < \pi$.

40. Obtain $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\, dx$, $x \neq \frac{k\pi}{2}$, $\tan x > 0$. **OR** Obtain $\int \frac{dx}{\sin^4 x + \cos^4 x}$.

SECTION D

- **Answer the following 41 to 50 questions as directed. Each question carries 3 marks. 30**

41. A is $(2\sqrt{2}, 0)$ and B is $(-2\sqrt{2}, 0)$. If $|AP - PB| = 4$, find the equation of locus of P.

42. Find the equation of the incircle of the triangle formed by the following lines.
$x = 2$, $4x + 3y = 5$ and $4x - 3y + 13 = 0$.

OR

Get the equation of the circle that passes through the origin and that cuts chords of length 5 on the lines $y = \pm x$.

43. Prove by vectors, that if the median on the base of a triangle is also altitude on the base, the triangle is isosceles.

OR

There are two forces $(2, 5, 6)$ and $(-1, 2, 1)$ that act on a particle and as a result of which the particle moves from A $(4, -3, -2)$ and B $(6, 1, -3)$. Find the work done.

44. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect each other and also find the point of intersection.

45. Obtain the equation of a plane that passes through the points $(2, 3, -4)$ and $(1, -1, 3)$, and that is parallel to X - axis.

46. Find $\lim_{x \to e^3} \frac{\log x - 3}{x - e^3}$.

47. Prove that of all the rectangles having the same area, the square has minimum perimeter.

OR

If $y = ax^3 + bx^2 + cx + 5$ touches X - axis at $(-2, 0)$ and the slope of the tangent where it meets Y - axis is 3, then find a, b and c.

48. Evaluate : $\int_0^1 \frac{\log(1+x)}{(1+x)^2}\, dx$.

49. Find the area of the region bounded by the curve $y = 2\sqrt{1 - x^2}$ and X - axis. **OR**

Evaluate : $\int_2^3 e^{-x}\, dx$ as the limit of a sum.

50. Solve the differential equation $x\,dy + y\,dx = xy\,dx$, $y(1) = 1$.

SECTION E

- **Answer the following 51 to 54 questions. Each question carries 5 marks. 20**

51. The equation of the line containing one of the sides of an equilateral triangle is $x + y = 2$ and one of the vertices of the triangle is $(2, 3)$. Find the equations of lines containing the remaining sides of the triangle.

OR

A is $(1, 3)$ in ΔABC and the lines $x - 2y + 1 = 0$ and $y - 1 = 0$ contain two medians of the triangle. Find the co - ordinates of B and C.

52. Find $\lim_{x \to 1} \frac{x^n - 1 - n(x-1)}{(x-1)^2}$, $x \neq 1$.

53. If $y = \log(1 + \sin x)$, then prove that $e^y \cdot \frac{d^2y}{dx^2} + 1 = 0$.

54. Evaluate : $\int \left(\frac{2007x + 2008}{2008x + 2007} \right) dx$ **OR** Evaluate : $\int \frac{dx}{\sin x + \sec x}$.