Instructions :

1. Answer all questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

SECTION - A

- Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question. **15**

1. Find the angle between the lines x = 3 and y = 5.

   (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

2. If the lines 5x - Ky - 7 = 0 and 2x + 3y + 5 = 0 are mutually perpendicular, then the value of K = ?

   (a) $-\frac{10}{3}$ (b) $\frac{10}{3}$ (c) $-\frac{15}{2}$ (d) $\frac{15}{2}$

3. How many tangents can be drawn from the origin (0, 0) to the curve $x^2 + y^2 = 25$?

   (a) 1 (b) 0 (c) 2 (d) 1 or 2

4. If $x^2 = -16y$, then find the equation of the tangent which is perpendicular to Y - axis.

   (a) x = 0 (b) y = 0 (c) x = 4 (d) x = - 4

5. Find the direction cosines of the vector 2i + 2j + k.

   (a) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ (b) - 2, 2, 1 (c) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ (d) 1, 1, 1

6. Find the eccentricity of hyperbola $3x^2 - 12y^2 = 36$.

   (a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $-\frac{\sqrt{5}}{\sqrt{2}}$ (d) $\frac{\sqrt{5}}{2}$

7. Find the projection of vector (1, 1) on the direction (3, 4).

   (a) $\frac{7}{5}$ (1, 1) (b) $\frac{7}{5}$ (3, 4) (c) $\frac{7}{25}$ (d) none of these

8. Find the rate of change of volume of a sphere having radius r with respect to its surface area.

   (a) r (b) 4r (c) $\frac{r}{2}$ (d) $\frac{r}{3}$

9. Find the centre of the sphere $|r^2| - r.(2, 1, 3) - 2 = 0$.

(a) $(-4, -2, -6)$ (b) $\left(1, \frac{1}{2}, \frac{3}{2}\right)$ (c) $(2, 1, 3)$ (d) $(0, 0, 0)$

10. $\lim_{x \to \infty} \frac{\sin\left[\tan\left(\sin\frac{5x}{6}\right)\right]}{x} = ?$

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) 1 (d) none of these

11. $\frac{d}{dx}\left|x - \frac{x^2}{2!}\right| + \frac{d}{dx}\left|(x+1) - \frac{2x^3}{3!}\right| = ?\ (x \neq -1)$

(a) $1 + x$ (b) $1 - x^2$ (c) $(1 + x)^{-1}$ (d) none of these

12. $\int \sin x . e^{\cos x}\, dx = ...$

(a) $e^{\cos x} + c$ (b) $e^{\sin x} + c$ (c) $-e^{\cos x} + c$ (d) none of these

13. $\int_{-1}^{1} \log\left(\frac{2 - x}{2 + x}\right) dx = ...$

(a) 1 (b) 0 (c) 2 (d) -2

14. Find the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\left(\frac{dy}{dx}\right)^3 + 4 = 0$.

(a) 2, 2 (b) 1, 3 (c) 3, 1 (d) 2, 3

15. Find the perpendicular distance from $P(4, -5, 3)$ to $r = (5, -2, 6) + K(3, -4, 5)$, $K \in R$.

(a) $\frac{\sqrt{547}}{5}$ (b) $\sqrt{\frac{457}{36}}$ (c) $\sqrt{\frac{457}{25}}$ (d) $\frac{\sqrt{547}}{6}$

SECTION B

- **Answer the following 16 to 30 questions. Each question carries one mark.** **15**

16. If $a, b, c \in R$ and if $a + b + c = 0$, prove that the lines $ax + by + c = 0$ pass through a fixed point $(a^2 + b^2 \neq 0)$. Also find that point.

17. Obtain the equation of a circle touching both axes and having its centre at $(4, -3)$.

18. Show that for every value t, the point $\left(\frac{a(1 - t^2)}{1 + t^2}, \frac{2bt}{1 + t^2} \right)$ lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

19. Find the projection of vector i on the direction of vector j.

20. If a particle moves from origin to (1, 1, 1) by application of forces (4, 1, - 3) and (3, 1, - 1), then find the work done.

21. Find the normal of a plane x + 2y + 3z = 6.

22. Find the sphere for which (1, 1, 2) and (2, 2, 1) are the extremities of its diameter.

23. Change N (- 2, 1) into the modulus form.

24. If $x^2 + y^2 = a^2$, then find $\frac{dy}{dx}$.

OR

Find $\frac{d}{dx}(m \cos^{-1} x)$.

25. If $f'(x) = \sqrt{x^2 + 7}$, $\forall\ x \in R$ and f (3) = 5, then find the approximate value of f (3.01).

26. Find $\int \frac{(2x + 1)^2}{x^2}\, dx$.

27. Evaluate $\int_0^{\frac{\pi}{4}} \sec x\, dx$.

28. Find the area of the region bounded by the curve xy = 16, X - axis and lines x = 4 and x = 8.

29. Find the differential equation of the curve $x^2 + y^2 = a^2$.

30. Get the equation of the tangent at (7, 7) to the parabola $y^2 = 7x$. **OR**

Get the tangent to $y^2 = 12x$ at the point t = 2. Also find the point of contact.

SECTION C

- **Answer the following 31 to 40 questions as directed. Each question carries two marks. 20**

31. Prove that (2a, 4a), (2a, 6a) and ($2a + \sqrt{3}\,a$, 5a) are the vertices of an equilateral triangle. ($a \neq 0$).

OR

O is (0, 0) and A is (- 2, - 3). Find the equation of the locus of P (x, y), if OP : AP = 5 : 3.

32. Find the equations of tangents to $y^2 = 4ax$ that makes an angle of measure $\frac{\pi}{3}$ with X - axis.

33. Find the tangents to the ellipse $3x^2 + 4y^2 = 12$ that are parallel to the line $3x + y - 2 = 0$.

34. Obtain the equation of the hyperbola passing through (5, - 2) having length of transverse axis equal to 7.

OR

Find the measure of angle between the asymptotes of $3x^2 - 2y^2 = 1$.

35. If $\bar{x}$ and $\bar{y}$ are unit vectors and $\bar{x} \cdot \bar{y} = 0$, then prove that $|\bar{x} + \bar{y}| = \sqrt{2}$.

36. Find the volume of the tetrahedron V - ABC, if V is (3, 2, - 4), A is (4, 3, 3), B is (3, 2, 1) and C is (1, 2, - 1).

OR

Find the area of Δ ABC, given that A, B, C are (1, 2, 1), (1, 3, 1) and (3, 4, 5) respectively.

37. Find $\lim_{h \to 0} \left[\frac{1}{\sqrt[3]{ah^3 + h^4}} \cdot \frac{1}{\sqrt[3]{ah^3}} \right]$.

38. If $y = \tan^{-1}\frac{5x}{1 - 6x^2}$, then find $\frac{dy}{dx}$.

OR

Differentiate $\tan^{-1}\frac{2x}{1 - x^2}$ with respect to $\cos^{-1}\frac{1 - x^2}{1 + x^2}$, where $0 < x < 1$.

39. Prove that of all the rectangles having the same area, the square has minimum perimeter.

40. Evaluate $\int \sin^3 (2x + 1)\, dx$.

SECTION D

- **Answer the following 41 to 50 questions as directed. Each question carries 3 marks. 30**

41. Find the fourth vertex of a parallelogram if the other three vertices are (1, 2), (- 2, 3) and (3, - 1).

42. Get the equation of the circle passing through (1, 0), (0, - 6) and (3, 4).

OR

If circles $x^2 + y^2 + 2gx + a^2 = 0$ and $x^2 + y^2 + 2fy + a^2 = 0$ touch each other externally, prove that $g^{-2} + f^{-2} = a^{-2}$.

43. If (a, 1, 1), (1, b, 1) and (1, 1, c) are co - planar, prove that $\frac{1}{1 - a} + \frac{1}{1 - b} + \frac{1}{1 - c} = 1$.

44. Prove that the lines $\frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4}$ and $\frac{x - 4}{5} = \frac{y - 1}{2} = z$ intersect and find the point of intersection.

OR

Find the point of intersection of the lines $r = (1, 1, - 1) + K (3, - 1, 0)$ and $r = (4, 0, - 1) + K (2, 0, 3)$, $k \in R$.

45. Get the equation of the plane through (1, 2, 3) and (3, - 1, 2) and perpendicular to the plane $x + 3y + 2z = 7$.

OR

Get the equation of the plane through (1, 2, - 1) and perpendicular to the line $\frac{x - 1}{3} = \frac{y - 2}{1} = \frac{z - 3}{2}$.

46. Divide 64 into two parts such that the sum of their cubes is minimum.

47. Evaluate $\int_0^{\frac{\pi}{2}} x^2 \cos 2x \, dx$.

48. Find the area of a region bounded by the curves $y = 1 - x^2$ and $y = x^2 - 1$.

49. A particle moves with constant acceleration and distance traveled by a particle during 10th and 12th second is 600 m and 720 m respectively. Find the initial velocity of the particle.

50. Solve $\cos (x - y) dy = dx$.

SECTION E

- **Answer the following 51 to 54 questions. Each question carries 5 marks. 20**

51. A is (1, 3) in ΔABC and the lines $x - 2y + 1 = 0$ and $y - 1 = 0$ contain two of the medians of the triangle. Find the co - ordinates of B and C.

52. Find $\lim_{x \to 0} \frac{(1 + mx)^n - (1 + nx)^m}{x^2}$, $m, n \in N$.

OR

If $f(x) = \begin{cases} 3x + 1, & x < 3 \\ Kx - 26, & 3 < x < 5 \\ x^2 + a, & x \geq 5 \end{cases}$ is continuous, then find K and a.

53. If $x\sqrt{1 - y^2} + y\sqrt{1 - x^2} = a$ and $|x| < 1$ and $|y| < 1$, then find $\frac{dy}{dx}$.

54. Evaluate : $\int \frac{\sin x}{\sqrt{1 + \sin x}} dx$, $x \in (0, \pi)$ **OR** Evaluate : $\int \frac{dx}{\sin x + \sec x}$.