

Instructions :

1. Answer all questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

**SECTION - A**

- Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question. 15

1. Find the angle between the lines  $x = 3$  and  $y = 5$ .

(a)  $\frac{\pi}{2}$       (b)  $\frac{\pi}{3}$       (c)  $\frac{\pi}{6}$       (d)  $\frac{\pi}{4}$

2. If the lines  $5x - Ky - 7 = 0$  and  $2x + 3y + 5 = 0$  are mutually perpendicular, then the value of  $K = ?$

(a)  $-\frac{10}{3}$       (b)  $\frac{10}{3}$       (c)  $-\frac{15}{2}$       (d)  $\frac{15}{2}$

3. How many tangents can be drawn from the origin  $(0, 0)$  to the curve  $x^2 + y^2 = 25$ ?

(a) 1      (b) 0      (c) 2      (d) 1 or 2

4. If  $x^2 = -16y$ , then find the equation of the tangent which is perpendicular to Y - axis.

(a)  $x = 0$       (b)  $y = 0$       (c)  $x = 4$       (d)  $x = -4$

5. Find the direction cosines of the vector  $2i + 2j + k$ .

(a)  $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$       (b)  $-2, 2, 1$       (c)  $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$       (d)  $1, 1, 1$

6. Find the eccentricity of hyperbola  $3x^2 - 12y^2 = 36$ .

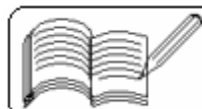
(a)  $\frac{3}{4}$       (b)  $-\frac{3}{4}$       (c)  $-\frac{\sqrt{5}}{\sqrt{2}}$       (d)  $\frac{\sqrt{5}}{2}$

7. Find the projection of vector  $(1, 1)$  on the direction  $(3, 4)$ .

(a)  $\frac{7}{5} (1, 1)$       (b)  $\frac{7}{5} (3, 4)$       (c)  $\frac{7}{25}$       (d) none of these

8. Find the rate of change of volume of a sphere having radius  $r$  with respect to its surface area.

(a)  $r$       (b)  $4r$       (c)  $\frac{r}{2}$       (d)  $\frac{r}{3}$



9. Find the centre of the sphere  $|r^2| - r \cdot (2, 1, 3) - 2 = 0$ .

- (a)  $(-4, -2, -6)$  (b)  $\left(1, \frac{1}{2}, \frac{3}{2}\right)$  (c)  $(2, 1, 3)$  (d)  $(0, 0, 0)$

10.  $\lim_{x \rightarrow \infty} \frac{\sin \left[ \tan \left( \sin \frac{5x}{6} \right) \right]}{x} = ?$

- (a)  $\frac{6}{5}$  (b)  $\frac{5}{6}$  (c) 1 (d) none of these

11.  $\frac{d}{dx} \left| x - \frac{x^2}{2!} \right| + \frac{d}{dx} \left| (x+1) - \frac{2x^3}{3!} \right| = ? \quad (x \neq -1)$

- (a)  $1 + x$  (b)  $1 - x^2$  (c)  $(1 + x)^{-1}$  (d) none of these

12.  $\int \sin x \cdot e^{\cos x} dx = \dots$

- (a)  $e^{\cos x} + c$  (b)  $e^{\sin x} + c$  (c)  $-e^{\cos x} + c$  (d) none of these

13.  $\int_{-1}^1 \log \left( \frac{2-x}{2+x} \right) dx = \dots$

- (a) 1 (b) 0 (c) 2 (d) -2

14. Find the order and degree of the differential equation  $\left( \frac{d^2y}{dx^2} \right)^2 + 3 \left( \frac{dy}{dx} \right)^3 + 4 = 0$ .

- (a) 2, 2 (b) 1, 3 (c) 3, 1 (d) 2, 3

15. Find the perpendicular distance from  $P(4, -5, 3)$  to  $r = (5, -2, 6) + K(3, -4, 5)$ ,  $K \in \mathbb{R}$ .

- (a)  $\frac{\sqrt{547}}{5}$  (b)  $\sqrt{\frac{457}{36}}$  (c)  $\sqrt{\frac{457}{25}}$  (d)  $\frac{\sqrt{547}}{6}$

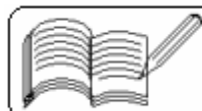
### SECTION B

• Answer the following 16 to 30 questions. Each question carries one mark.

15

16. If  $a, b, c \in \mathbb{R}$  and if  $a + b + c = 0$ , prove that the lines  $ax + by + c = 0$  pass through a fixed point ( $a^2 + b^2 \neq 0$ ). Also find that point.

17. Obtain the equation of a circle touching both axes and having its centre at  $(4, -3)$ .



18. Show that for every value  $t$ , the point  $\left( \frac{a(1-t^2)}{1+t^2}, \frac{2bt}{1+t^2} \right)$  lies on the ellipse

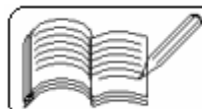
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

19. Find the projection of vector  $i$  on the direction of vector  $j$ .
20. If a particle moves from origin to  $(1, 1, 1)$  by application of forces  $(4, 1, -3)$  and  $(3, 1, -1)$ , then find the work done.
21. Find the normal of a plane  $x + 2y + 3z = 6$ .
22. Find the sphere for which  $(1, 1, 2)$  and  $(2, 2, 1)$  are the extremities of its diameter.
23. Change  $N(-2, 1)$  into the modulus form.
24. If  $x^2 + y^2 = a^2$ , then find  $\frac{dy}{dx}$ .

OR

Find  $\frac{d}{dx}(m \cos^{-1} x)$ .

25. If  $f'(x) = \sqrt{x^2 + 7}$ ,  $\forall x \in \mathbb{R}$  and  $f(3) = 5$ , then find the approximate value of  $f(3.01)$ .
26. Find  $\int \frac{(2x+1)^2}{x^2} dx$ .
27. Evaluate  $\int_0^{\frac{\pi}{4}} \sec x dx$ .
28. Find the area of the region bounded by the curve  $xy = 16$ , X-axis and lines  $x = 4$  and  $x = 8$ .
29. Find the differential equation of the curve  $x^2 + y^2 = a^2$ .



30. Get the equation of the tangent at  $(7, 7)$  to the parabola  $y^2 = 7x$ . OR

Get the tangent to  $y^2 = 12x$  at the point  $t = 2$ . Also find the point of contact.

**SECTION C**

- Answer the following 31 to 40 questions as directed. Each question carries two marks. 20

31. Prove that  $(2a, 4a)$ ,  $(2a, 6a)$  and  $(2a + \sqrt{3}a, 5a)$  are the vertices of an equilateral triangle. ( $a \neq 0$ ).

OR

O is  $(0, 0)$  and A is  $(-2, -3)$ . Find the equation of the locus of  $P(x, y)$ , if  $OP : AP = 5 : 3$ .

32. Find the equations of tangents to  $y^2 = 4ax$  that makes an angle of measure  $\frac{\pi}{3}$  with X - axis.

33. Find the tangents to the ellipse  $3x^2 + 4y^2 = 12$  that are parallel to the line  $3x + y - 2 = 0$ .

34. Obtain the equation of the hyperbola passing through  $(5, -2)$  having length of transverse axis equal to 7.

OR

Find the measure of angle between the asymptotes of  $3x^2 - 2y^2 = 1$ .

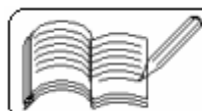
35. If  $\bar{x}$  and  $\bar{y}$  are unit vectors and  $\bar{x} \cdot \bar{y} = 0$ , then prove that  $|\bar{x} + \bar{y}| = \sqrt{2}$ .

36. Find the volume of the tetrahedron  $V - ABC$ , if V is  $(3, 2, -4)$ , A is  $(4, 3, 3)$ , B is  $(3, 2, 1)$  and C is  $(1, 2, -1)$ .

OR

Find the area of  $\Delta ABC$ , given that A, B, C are  $(1, 2, 1)$ ,  $(1, 3, 1)$  and  $(3, 4, 5)$  respectively.

37. Find  $\lim_{h \rightarrow 0} \left[ \frac{1}{\sqrt[3]{ah^3 + h^4}} - \frac{1}{\sqrt[3]{ah^3}} \right]$ .



38. If  $y = \tan^{-1} \frac{5x}{1 - 6x^2}$ , then find  $\frac{dy}{dx}$ .

OR

Differentiate  $\tan^{-1} \frac{2x}{1 - x^2}$  with respect to  $\cos^{-1} \frac{1 - x^2}{1 + x^2}$ , where  $0 < x < 1$ .

39. Prove that of all the rectangles having the same area, the square has minimum perimeter.

40. Evaluate  $\int \sin^3 (2x + 1) dx$ .

**SECTION D**

• Answer the following 41 to 50 questions as directed. Each question carries 3 marks. 30

41. Find the fourth vertex of a parallelogram if the other three vertices are (1, 2), (-2, 3) and (3, -1).

42. Get the equation of the circle passing through (1, 0), (0, -6) and (3, 4).

OR

If circles  $x^2 + y^2 + 2gx + a^2 = 0$  and  $x^2 + y^2 + 2fy + a^2 = 0$  touch each other externally, prove that  $g^2 + f^2 = a^2$ .

43. If (a, 1, 1), (1, b, 1) and (1, 1, c) are co-planar, prove that  $\frac{1}{1 - a} + \frac{1}{1 - b} + \frac{1}{1 - c} = 1$ .

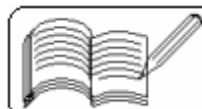
44. Prove that the lines  $\frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4}$  and  $\frac{x - 4}{5} = \frac{y - 1}{2} = z$  intersect and find the point of intersection.

OR

Find the point of intersection of the lines  $r = (1, 1, -1) + K(3, -1, 0)$  and  $r = (4, 0, -1) + K(2, 0, 3)$ ,  $k \in R$ .

45. Get the equation of the plane through (1, 2, 3) and (3, -1, 2) and perpendicular to the plane  $x + 3y + 2z = 7$ .

OR



Get the equation of the plane through ( 1, 2, - 1 ) and perpendicular to the line

$$\frac{x - 1}{3} = \frac{y - 2}{1} = \frac{z - 3}{2}.$$

46. Divide 64 into two parts such that the sum of their cubes is minimum.

47. Evaluate  $\int_0^{\frac{\pi}{2}} x^2 \cos 2x \, dx$ .

48. Find the area of a region bounded by the curves  $y = 1 - x^2$  and  $y = x^2 - 1$ .

49. A particle moves with constant acceleration and distance traveled by a particle during 10th and 12th second is 600 m and 720 m respectively. Find the initial velocity of the particle.

50. Solve  $\cos (x - y) \, dy = dx$ .

### SECTION E

• Answer the following 51 to 54 questions. Each question carries 5 marks. 20

51. A is ( 1, 3 ) in  $\Delta ABC$  and the lines  $x - 2y + 1 = 0$  and  $y - 1 = 0$  contain two of the medians of the triangle. Find the co - ordinates of B and C.

52. Find  $\lim_{x \rightarrow 0} \frac{(1 + mx)^n - (1 + nx)^m}{x^2}$ ,  $m, n \in \mathbb{N}$ .

OR

If  $f(x) = \begin{cases} 3x + 1, & x < 3 \\ Kx - 26, & 3 < x < 5 \\ x^2 + a, & x \geq 5 \end{cases}$  is continuous, then find K and a.

53. If  $x\sqrt{1 - y^2} + y\sqrt{1 - x^2} = a$  and  $|x| < 1$  and  $|y| < 1$ , then find  $\frac{dy}{dx}$ .

54. Evaluate :  $\int \frac{\sin x}{\sqrt{1 + \sin x}} \, dx$ ,  $x \in (0, \pi)$  OR Evaluate :  $\int \frac{dx}{\sin x + \sec x}$ .

