

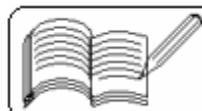
## Instructions :

1. Answer all questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

## SECTION - A

- Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question. 15

1. For  $\Delta ABC$ , A is (1, 2), B is (3, 4) and C is on X-axis. If the centroid of  $\Delta ABC$  is on Y-axis, then find the coordinates of C.  
(a) (-4, 0)      (b) (2, 0)      (c) (0, -4)      (d) none of these
2. Find the equation of the perpendicular bisector of  $\overline{AB}$ , where A is (2, 2) and B is (4, 2).  
(a)  $y - 3 = 0$       (b)  $y + 3 = 0$       (c)  $x - 3 = 0$       (d)  $x - 2 = 0$
3. Find the radius of a circle touching X-axis and having its centre at (4, -3)  
(a) 4      (b) 3      (c) 5      (d) none of these.
4. There is a point on the parabola  $y^2 = 2x$ , whose x-coordinate is two times the y-coordinate. Find the point.  
(a) (4, 2)      (b) (6, 3)      (c) (4, 8)      (d) (8, 4)
5. Find the measure of the angle between the asymptotes of  $x^2 - y^2 = 16$ .  
(a)  $\frac{\pi}{2}$       (b)  $\frac{\pi}{3}$       (c)  $\frac{\pi}{4}$       (d) none of these
6. If  $|\vec{a}| = 5$ ,  $|\vec{b}| = 3$  and  $|\vec{a} - \vec{b}| = 4$ , then find  $\vec{a} \cdot \vec{b}$ .  
(a) -9      (b) 0      (c) 9      (d) none of these
7. Force  $\vec{i} + \vec{j} + \vec{k}$  is applied at B (1, 2, 3). Find the torque around A (-1, 2, 0) and its magnitude.  
(a)  $\sqrt{14}$       (b) (-3, 1, 2)      (c) (3, -1, -2)      (d) none of these
8. Find the measure of the angle between the planes  $ax + by + d = 0$  and  $z = 0$ , ( $a^2 + b^2 \neq 0$ ).  
(a)  $\frac{\pi}{3}$       (b)  $\cos^{-1} \frac{d}{\sqrt{a^2 + b^2}}$       (c)  $\frac{\pi}{4}$       (d)  $\frac{\pi}{2}$

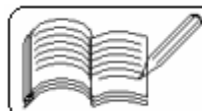


9. Find  $\lim_{x \rightarrow 0} \frac{\sin x}{|x|}$ .  
 (a) -1 (b) 1 (c) 0 (d) limit does not exist
10. Find  $\frac{d}{dx} \left( e^{(\sin^{-1}x + \cos^{-1}x)} \right), |x| < 1$   
 (a)  $e^{(\sin^{-1}x + \cos^{-1}x)}$  (b)  $\frac{e^{(\sin^{-1}x + \cos^{-1}x)}}{\sqrt{1-x^2}}$  (c) 0 (d) none of these
11. Find  $\int e^x \sec x (1 + \tan x) dx$ .  
 (a)  $e^x \tan x + c$  (b)  $e^x \sec x + c$  (c)  $e^x \tan^2 x + c$  (d) none of these
12. If  $f$  is an even function and  $\int_{-a}^a f(x) dx = 2$ , then find  $\int_0^a f(x) dx$ .  
 (a) 0 (b) 4 (c)  $\frac{a}{2}$  (d) 1
13. Find the area of the region bounded by the curve  $y = \cos x$ , X-axis and the lines  $x = 0, x = \pi$ .  
 (a) 2 (b) 1 (c) 4 (d) none of these
14. Obtain the order of differential equation  $3\sqrt{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}}$ .  
 (a) 1 (b) does not exist (c) 2 (d) 3
15. A ball is projected vertically upwards with speed 19.6 m/s. Find the time for maximum height.  
 (a) 2 seconds (b) 4 seconds (c) 19.6 seconds (d) none of these

<b>SECTION B</b>
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• Answer the following 16 to 30 questions. Each question carries one mark. 15

16. For  $A(-2, 3)$  and  $B(3, 0)$ , find the ratio in which the Y-axis divides  $\overline{AB}$  from A-side.



17. Obtain the equation of a circle given that its area is  $49\pi$  and the equations of lines containing two of the diameters of the circle are  $2x - 3y + 12 = 0$  and  $x + 4y - 5 = 0$ .
18. Find the equation of a parabola which passes through  $(2, 3)$  and is symmetric about X-axis. The vertex of the parabola is at the origin.
19. Find the eccentricity of the ellipse, the length of whose latus - rectum is half the length of major axis.

OR

Obtain the equation of the auxiliary circle and director circle of the ellipse

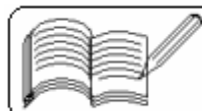
$$\frac{x^2}{16} + \frac{y^2}{9} = 1.$$

20. In  $\mathbb{R}^2$ , find the unit vector orthogonal to  $(3, 4)$ .
21. Find the volume of the tetrahedron V - ABC if V is  $(3, 2, -4)$ , A  $(4, 3, 3)$  B  $(3, 2, 1)$  and C  $(1, 2, -1)$ .
22. Find the equation of the line through  $(2, 3, 4)$  and parallel to the line  $\frac{x-1}{3} = \frac{2-y}{-5} = \frac{z-10}{15}$ .
23. Find the centre and radius of the sphere  $x^2 + y^2 + z^2 - 4x - 4y + 8z - 1 = 0$ .
24. If  $f(x) = \log_x 7$ , then find  $f'(7)$ .
25. Radius of a circular metal plate when heated increases by 2%. If its radius is 10 cm, find the increase in its area

OR

Apply Rolle's theorem to  $f(x) = \sin x + \cos x + 1$ ,  $x \in \left[ \pi, \frac{3\pi}{2} \right]$  and find c.

26. Using the formula  $\int [f(x) + f'(x)]e^x dx = e^x f(x) + c$ ,  
find  $\int \left( \log x + \frac{1}{x^2} \right) e^x dx$ ,  $x > 0$  OR Find  $\int \frac{e^x + 1}{e^x - 1} dx$ .
27. If  $\int_0^k \frac{dx}{2 + 8x^2} = \frac{\pi}{16}$ , then find k.
28. Solve  $\sec^2 x \cdot \tan y dx + \sec^2 y \cdot \tan x dy = 0$ .



29. If initial velocity of projectile is 28 m/s and horizontal range is 40 m, find the measure of angle of projection.
30. A particle moves on a line and its distance from a fixed point at time  $t$  is  $x$ , where  $x = 4t^2 + 2t$ . Find velocity and acceleration at  $t = \frac{1}{2}$ .

## SECTION C

- Answer the following 31 to 40 questions as directed. Each question carries two marks. 20

31. Find parametric equations of the lines passing through  $A(3, -1)$ ,  $B(0, 3)$ . Also write  $\vec{BA} - \vec{AB}$  as sets.

OR

If the distances from the origin to the lines  $x \sec \theta + y \operatorname{cosec} \theta = a$  and  $x \cos \theta - y \sin \theta = a \cos 2\theta$  are  $p$  and  $p'$  respectively, prove that  $4p^2 + p'^2 = a^2$ .

32. If the line  $3x + 4y + 16 = 0$  is tangent to the parabola  $y^2 = Kx$ , find  $K$  and the point of contact.

OR

One end - point of a focal chord of the parabola  $y^2 = 16x$  is  $(4, 8)$ . Find the other end - point and the length of the focal chord.

33. If a tangent to  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  intersects the major axis at  $T$  and minor axis at  $N$ , and if

$C$  is the centre, then prove that  $\frac{a^2}{CT^2} + \frac{b^2}{CN^2} = 1$ .

34. If the chord of the hyperbola joining  $P(\theta)$  and  $Q(\phi)$  on the hyperbola subtends a right angle at the centre  $C(0, 0)$ , then prove that  $a^2 + b^2 \sin \theta \cdot \sin \phi = 0$ .

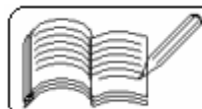
OR

Obtain the equations of the tangents to the hyperbola  $5x^2 - y^2 = 5$  from the point  $(0, 2)$ .

35. Find a unit vector orthogonal to  $(2, 1, 1)$  and  $(1, 2, 3)$ .

36. If  $\vec{x} \perp \vec{y}$  and  $\vec{x}$  and  $\vec{y}$  are unit vectors, show that  $|\vec{x} \times \vec{y}|$  is also a unit vector.

37. Get the radius of the circle that is formed by the intersection of the sphere  $x^2 + y^2 + z^2 = 25$  and the plane  $2x + 2y + z = 12$ .



38. If  $f(a) = 2$ ,  $f'(a) = 1$ ,  $g(a) = -1$  and  $g'(a) = 2$ , then find

$$\lim_{x \rightarrow a} \frac{f(x) \cdot g(a) - g(x) \cdot f(a)}{x - a}.$$

OR

If  $x^2 - y^2 = 1$ , then prove that  $y^3 \frac{d^2y}{dx^2} + 1 = 0$ .

39.  $(2, 3)$  lies on  $y^2 = ax^3 + b$ . Slope of tangent at  $(2, 3)$  is 4. Find  $a$  and  $b$ .

40. Evaluate :  $\int \frac{e^x (1+x)}{\sin^2(xe^x)} dx$ .

### SECTION D

- Answer the following 41 to 50 questions as directed. Each question carries 3 marks. 30

41. If  $P(at^2, 2at)$ ,  $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$  and  $S(a, 0)$  are the points, show that  $\frac{1}{SP} + \frac{1}{SQ}$  is independent of  $t$

OR

Find the incentre of the triangle, whose vertices are  $(4, 1)$ ,  $(1, 5)$  and  $(-2, 1)$ .

42. Get the equation of the circle touching both the axes and also touching the line  $3x + 4y - 6 = 0$  in the first quadrant.

OR

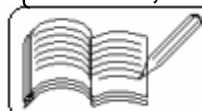
Show that the circles  $x^2 + y^2 + 6x + 2y - 90 = 0$  and  $x^2 + y^2 - 34x - 28y + 260 = 0$  touch each other externally.

43. Forces  $(2, 5, 6)$  and  $(-1, 2, 1)$  act on a particle as a result of which the particle moves from  $A(4, -3, -2)$  to  $B(6, 1, -3)$ . If the unit of force is newton and distance is measured in meters, find the work done.

44. Find the perpendicular distance from  $A(1, 0, 3)$  to the line  $\vec{r} = (4, 7, 1) + k(1, 2, -2)$ ,  $k \in \mathbb{R}$ . Also find the foot of the perpendicular.

45. Get the equation of the plane passing through  $(1, 2, 3)$  and  $(3, -1, 2)$  and perpendicular to the plane  $x + 3y + 2z = 7$ .

46.  $f(x) = \begin{cases} 3ax + b, & x > 1 \\ 11, & x = 1 \\ 5ax - 2b, & x < 1 \end{cases}$ .  $f$  is continuous at  $x = 1$ . Find  $a$  and  $b$ .



47. Two trains start from the same place. One travels towards south at a speed of 50 km/h and another travels towards west at a speed of 60 km/h. Find the distance between them after two hours.

OR

Divide 64 into two parts such that the sum of their cubes is minimum.

48. Obtain  $\int_1^4 4^x dx$  as the limit of a sum.

49. Obtain  $\int_0^{\frac{\pi}{4}} \frac{\sin 2\theta}{\sin^4 \theta + \cos^4 \theta} d\theta$ .

50. Solve differential equation  $x \frac{dy}{dx} = x + y$ .

**SECTION E**

- Answer the following 51 to 54 questions. Each question carries 5 marks. 20

51. A line passes through (4, -2) and the length of the perpendicular segment from the origin to this line is 2. Find the equation of the line.

OR

Find the equation of the line passing through (2, 3) and containing a line - segment of length  $\frac{2\sqrt{2}}{3}$  between the lines  $2x + y = 3$  and  $2x + y = 5$ .

52. Find  $\lim_{x \rightarrow 0} \frac{1 - \cos x \cdot \sqrt{\cos 2x}}{x^2}$ .

53. If  $x = a(\cos \theta + \theta \sin \theta)$ ,  $y = a(\sin \theta - \theta \cos \theta)$ , then prove that

$$y_2 = \frac{\sec^3 \theta}{a\theta}, \theta \in \left(0, \frac{\pi}{2}\right), a \neq 0.$$

54. Evaluate  $\int \frac{dx}{x^4 + 1}$  OR Evaluate  $\int \sqrt{\frac{x-1}{x-3}} dx$ , where  $x > 3$ .

