

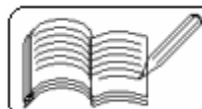
Instructions :

1. Answer all questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

SECTION - A

- Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question.

1. $d[(17, -8), (1-7, -3)] = ?$
(a) -5 (b) 11 (c) 5 (d) -11
2. The Cartesian equation of the line passing through (5, 6) and (-3, 6) is ...
(a) $y - 6 = 0$ (b) $y + 6 = 0$ (c) $x - 5 = 0$ (d) $x + 3 = 0$
3. The equation of the circle touching the y-axis and having its centre at (3, -4) is ...
(a) $x^2 + y^2 + 6x + 8y + 16 = 0$ (b) $x^2 + y^2 - 6x + 8y + 9 = 0$
(c) $x^2 + y^2 - 6x - 8y + 9 = 0$ (d) $x^2 + y^2 - 6x + 8y + 16 = 0$
4. The end points of the latus-rectum for parabola $x^2 = -6y$ are
(a) $\left(\pm 3, -\frac{3}{2}\right)$ (b) $\left(-\frac{3}{2}, 3\right)$ (c) $\left(-\frac{3}{2}, -3\right)$ (d) $\left(\pm 3, \frac{3}{2}\right)$
5. Measure of the angle between asymptotes of $4x^2 - y^2 = 9$ is ...
(a) $\tan^{-1}\left(-\frac{4}{3}\right)$ (b) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$ (c) $\frac{\pi}{3}$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$
6. Which is a unit vector?
(a) $(\cos\alpha, 2\sin\alpha)$ (b) $(\sin\alpha, \cos\alpha)$ (c) $(1, -1)$ (d) $(2\cos\alpha, \sin\alpha)$
7. $\bar{x} = (1, -1)$ and $\bar{y} = (1, 0)$, then $\cos \bar{x} \wedge \bar{y} =$
(a) 1 (b) 0 (c) $\frac{1}{\sqrt{2}}$ (d) \bar{y}
8. Measure of the angle between $x + 2y + z = 1$ and $\bar{r} = (0, 0, 0) + k(2, 1, -1)$, $k \in \mathbb{R}$ is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$



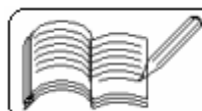
9. The plane $\vec{r} \cdot (2, -2, 1) = -12$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$, then the point of contact is
 (a) $(1, -4, 2)$ (b) $(-1, 4, -2)$ (c) $(-1, 4, 2)$ (d) none of these
10. $\lim_{x \rightarrow \frac{1}{4}} \frac{e^{4x} - e}{x - \frac{1}{4}} = ?$
 (a) $4e$ (b) $\frac{e}{4}$ (c) $-4e$ (d) $\log_e 4$
11. The derivative of $\sin^{-1}x$ with respect to $\cos^{-1}x$ is ...
 (a) 1 (b) -1 (c) 0 (d) none of these
12. Radius of a circular metal plate, when heated, increases by 2%. If its radius is 10 cm., then the increase in its area is ...
 (a) $4\pi \text{ cm}^2$ (b) $4\pi \text{ cm}$ (c) $20\pi \text{ cm}^2$ (d) $2\pi \text{ cm}^2$
13. $\int_{-1}^0 |x| dx =$
 (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) none of these
14. The degree and order of the differential equation $\frac{d^2y}{dx^2} = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}}$ are ...
 (a) 6 and 1 (b) 3 and 2 (c) 2 and 2 (d) 1 and 1
15. A body projected in vertical direction attains maximum height 50 m. Its velocity at 25 m height is
 (a) $7\sqrt{10} \text{ m/s}$ (b) $7\sqrt{10} \text{ m/s}^2$ (c) $-7\sqrt{10} \text{ m/s}$ (d) 490 m

SECTION B

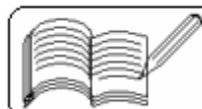
- Answer the following 16 to 30 questions. Each question carries one mark.

15

16. In which ratio does the x-axis divide the line segment joining A(3, 5) and B(2, 6)?
17. Obtain the equation of the circle which has a diagonal of rectangle formed by $x = 2$, $x = -2$, $y = 3$ and $y = 1$.
 OR
 Obtain the equation of a circle with radius $5/2$, if it passes through $(-1, 1)$ and $(-1, -4)$.



18. There is a point on the parabola $y^2 = 2x$ whose x - coordinate is two times the y - coordinate. If this point is not the vertex of the parabola, find the point.
19. Find the parametric equation of the director circle of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.
20. Find a unit vector orthogonal to both $(2, 2, 1)$ and $(3, 2, 2)$.
21. Find the projection of $(1, 1, 1)$ on $(2, 2, 1)$.
22. Find the perpendicular distance of the point $P(4, -5, 3)$ from the line $\frac{x - 5}{3} = \frac{y + 2}{-4} = \frac{z - 6}{5}$.
23. Find $\frac{d}{dx}(\sin^3 x^\circ)$ OR Find $\frac{d}{dx}(e^{-2006 \log_e x})$.
24. Evaluate $\int \frac{e^x}{\sqrt{2x^2 + 3}} dx$.
25. Find the area of the region bounded by the curve $y = \cos x$, x - axis and the lines $x = 0$ and $x = \pi$.
26. Evaluate $\int \tan^2 x \sec^2 x dx$ OR Evaluate $\int \frac{1}{9 + 4x^2} dx$.
27. Evaluate $\int_1^{4013} (\operatorname{cosec}^{-1} x + \sec^{-1} x) dx, |x| \geq 1$.
28. Obtain the differential equation representing all the lines of family $y = mx + c$ (where m and c are arbitrary constants).
29. If the distance of a particle executing rectilinear motion is x from a fixed point at time t , where $x = 2t^3 - 9t^2 + 12t + 8$, then when will the velocity become 0.
30. Two balls are thrown vertically upwards with velocities 19.6 m/s and 9.8 m/s. Find the height of the second ball, when the first ball attains maximum height.



SECTION C

- Answer the following 31 to 40 questions as directed. Each question carries two marks. 20

31. Prove by using slopes that A (12, 8) B (-2, 6) and C (6, 0) are the vertices of a right triangle.

OR

Find the equation of the perpendicular bisector of \overline{AB} where A is (-3, 2) and B is (7, 6).

32. For the parabola $x^2 = 12y$, find the area of the triangle whose vertices are the vertex of the parabola and two end - points of its latus - rectum.

33. If the end - points of a chord of the ellipse $b^2x^2 + a^2y^2 - a^2b^2 = 0$ have eccentric angles with measure α and β , then prove that the equation of the line containing the chord is

$$\frac{x}{a} \cos\left(\frac{\alpha + \beta}{2}\right) + \frac{y}{b} \sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha - \beta}{2}\right).$$

34. If the eccentricities of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1$ are e_1 and e_2 respectively, then prove that

$$e_1^2 + e_2^2 = e_1^2 \cdot e_2^2.$$

OR

If the chord of hyperbola joining P (α) and Q (β) on the hyperbola subtends a right angle at the centre C (0, 0), then prove that $a^2 + b^2 \sin\alpha \sin\beta = 0$.

35. Prove that: $[\bar{x} + \bar{y} \quad \bar{y} + \bar{z} \quad \bar{z} + \bar{x}] = 2[\bar{x} \quad \bar{y} \quad \bar{z}]$.

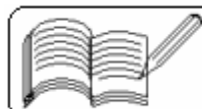
36. If $\bar{x}, \bar{y}, \bar{z}$ are coplanar vectors, then prove that $\bar{x} + \bar{y}, \bar{y} + \bar{z},$ and $\bar{z} + \bar{x}$ are coplanar.

OR

If $(\bar{x} + \bar{y}) \cdot (\bar{x} - \bar{y}) = 63$ and $|\bar{x}| = 8|\bar{y}|$, then find $|\bar{x}|$.

37. Get the radius of the circle that is the intersection of the sphere $x^2 + y^2 + z^2 = 49$ and the plane $2x + 3y - z = 5\sqrt{14}$.

38. If $x = a(1 - \cos\theta), y = a(\theta - \sin\theta), \theta \in [0, \pi], a \neq 0$, then find $\frac{d^2y}{dx^2}$.



39. Verify Rolle's theorem for $f(x) = \sin x + \cos x - 1$, $x \in \left[0, \frac{\pi}{2}\right]$. If it is applicable, find c .

OR

In which interval the function $f(x) = 5x^3 - 15x^2 - 120x + 3$ is increasing and in which interval is it decreasing?

40. Evaluate $\int \frac{\sin x}{1 + \sin x} dx$.

SECTION D

- Answer the following 41 to 50 questions as directed. Each question carries 3 marks. 30

41. A is $(2\sqrt{2}, 0)$ and B is $(-2\sqrt{2}, 0)$. If $|AP - PB| = 4$, then find the equation of locus of P.

OR

Origin is circumcentre of triangle with vertices

$A(x_1, x_1 \tan \theta_1)$, $B(x_2, x_2 \tan \theta_2)$ and $C(x_3, x_3 \tan \theta_3)$ ($0 < \theta_i < \pi/2$, $x_i > 0$, $i = 1, 2, 3$).

If the centroid of $\triangle ABC$ is (x, y) , prove that $\frac{y}{x} = \frac{\sin \theta_1 + \sin \theta_2 + \sin \theta_3}{\cos \theta_1 + \cos \theta_2 + \cos \theta_3}$.

42. If the equation $3x^2 + (3 - p)xy + qy^2 - 2px = 8pq$ represents a circle, find p and q . Also determine the centre and radius of the circle.

43. Forces measuring 5, 3 and 1 unit act in the direction $(6, 2, 3)$, $(3, -2, 6)$, $(2, -3, -6)$ respectively. As a result, the particle moves from $(2, -1, -3)$ to $(5, -1, 1)$. Find the resultant force and work done.

44. Find the vector and Cartesian equations of the line passing through $(1, 2, 3)$ and perpendicular to the two lines

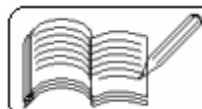
$$\vec{r} = (0, 0, 0) + K(1, 2, -1), K \in \mathbb{R} \text{ and } \frac{x-1}{3} = \frac{y}{2} = \frac{z}{6}. \quad \text{OR}$$

Find the measure of the angle between two lines, if their direction cosines l, m, n satisfy $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$.

45. Find the vector and Cartesian equations of the plane containing the lines

$$\vec{r} = (1, 2, 3) + K(2, 3, 4), K \in \mathbb{R} \text{ and } \frac{x-1}{1} = \frac{y}{3} = \frac{z-5}{4}.$$

46. Find $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{x - \cos(\sin^{-1}x)}{1 - \tan(\sin^{-1}x)}$.



47. Prove that if $x > 0$, then $\frac{x}{1+x^2} < \tan^{-1}x < x$.

48. Obtain $\int_0^{\frac{\pi}{2}} \sin x \, dx$ as the limit of a sum.

49. Prove that $\int_8^{27} \frac{dx}{x - x^{\frac{1}{3}}} = \frac{3}{2} \log \left(\frac{8}{3} \right)$.

50. Solve $xy \frac{dy}{dx} = y + 2$. If $y(2) = 0$, then find the particular solution of the given differential equation. **OR**

The population of a city increases at the rate of 3% per year. How many years will it take for the population to double?

SECTION E

• Answer the following 51 to 54 questions. Each question carries 5 marks. 20

51. A is $(-4, -5)$ in $\triangle ABC$ and the lines $5x + 3y - 4 = 0$ and $3x + 8y + 13 = 0$ contain two of the altitudes of the triangle. Find the coordinates of B and C.

52. If $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, $x \neq 0$, $f(0) = 1$, then prove that f is not continuous at $x = 0$.

OR

Find $\lim_{x \rightarrow 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$, $m, n \in \mathbb{N}$.

53. If $x = \sin t$ and $y = \sin pt$, then prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0$.

54. Evaluate $\int \frac{1}{1 + 5e^x + 6e^{2x}} dx$ **OR** Evaluate $\int \frac{\sec x}{1 + \operatorname{cosec} x} dx$.

