Time : 3 hours **02 - Mathematics - July 2006** **Marks : 100**

Instructions :

1. Answer all questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

SECTION - A

- Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question.

1. $d[(|7|, -8), (|-7|, -3)] = ?$
   (a) -5 (b) 11 (c) 5 (d) -11

2. The Cartesian equation of the line passing through (5, 6) and (- 3, 6) is …
   (a) $y - 6 = 0$ (b) $y + 6 = 0$ (c) $x - 5 = 0$ (d) $x + 3 = 0$

3. The equation of the circle touching the y - axis and having its centre at (3, - 4) is …
   (a) $x^2 + y^2 + 6x + 8y + 16 = 0$ (b) $x^2 + y^2 - 6x + 8y + 9 = 0$
   (c) $x^2 + y^2 - 6x - 8y + 9 = 0$ (d) $x^2 + y^2 - 6x + 8y + 16 = 0$

4. The end points of the latus - rectum for parabola $x^2 = -6y$ are
   (a) $\left(\pm 3, -\frac{3}{2}\right)$ (b) $\left(-\frac{3}{2}, 3\right)$ (c) $\left(-\frac{3}{2}, -3\right)$ (d) $\left(\pm 3, \frac{3}{2}\right)$

5. Measure of the angle between asymptotes of $4x^2 - y^2 = 9$ is …
   (a) $\tan^{-1}\left(-\frac{4}{3}\right)$ (b) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$ (c) $\frac{\pi}{3}$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

6. Which is a unit vector ?
   (a) $(\cos\alpha, 2\sin\alpha)$ (b) $(\sin\alpha, \cos\alpha)$ (c) $(1, -1)$ (d) $(2\cos\alpha, \sin\alpha)$

7. $\bar{x} = (1, -1)$ and $\bar{y} = (1, 0)$, then $\cos \bar{x} \wedge \bar{y} =$
   (a) 1 (b) 0 (c) $\frac{1}{\sqrt{2}}$ (d) $\bar{y}$

8. Measure of the angle between $x + 2y + z = 1$ and $\bar{r} = (0, 0, 0) + k(2, 1, -1), k \in R$ is
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

9. The plane $\bar{r} \cdot (2, -2, 1) = -12$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$, then the point of contact is

(a) (1, - 4, 2) (b) (- 1, 4, - 2) (c) (- 1, 4, 2) (d) none of these

10. $\lim\limits_{x \to \frac{1}{4}} \dfrac{e^{4x} - e}{x - \frac{1}{4}} = ?$

(a) 4e (b) $\frac{e}{4}$ (c) - 4e (d) $\log_e 4$

11. The derivative of $\sin^{-1}x$ with respect to $\cos^{-1}x$ is …

(a) 1 (b) - 1 (c) 0 (d) none of these

12. Radius of a circular metal plate, when heated, increases by 2 %. If its radius is 10 cm., then the increase in its area is …

(a) 4π cm^2 (b) 4π cm (c) $20\,\pi$ cm^2 (d) $2\,\pi$ cm^2

13. $\int_{-1}^{0} |x|\,dx =$

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) none of these

14. The degree and order of the differential equation $\dfrac{d^2y}{dx^2} = \left(1 + \left(\dfrac{dy}{dx}\right)^2\right)^{\frac{1}{2}}$ are …

(a) 6 and 1 (b) 3 and 2 (c) 2 and 2 (d) 1 and 1

15. A body projected in vertical direction attains maximum height 50 m. Its velocity at 25 m height is

(a) $7\sqrt{10}$ m/s (b) $7\sqrt{10}$ m/s^2 (c) $-7\sqrt{10}$ m/s (d) 490 m

SECTION B

- **Answer the following 16 to 30 questions. Each question carries one mark.** 15

16. In which ratio does the x – axis divide the line segment joining A (3, 5) and B (2, 6) ?

17. Obtain the equation of the circle which has a diagonal of rectangle formed by x = 2, x = - 2, y = 3 and y = 1.

OR

Obtain the equation of a circle with radius 5/2, if it passes through (- 1, 1) and (- 1, - 4).

18. There is a point on the parabola $y^2 = 2x$ whose x - coordinate is two times the y - coordinate. If this point is not the vertex of the parabola, find the point.

19. Find the parametric equation of the director circle of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

20. Find a unit vector orthogonal to both (2, 2, 1) and (3, 2, 2).

21. Find the projection of (1, 1, 1) on (2, 2, 1).

22. Find the perpendicular distance of the point P (4, - 5, 3) from the line $\frac{x - 5}{3} = \frac{y + 2}{-4} = \frac{z - 6}{5}$.

23. Find $\frac{d}{dx}(\sin^3 x^\circ)$ OR Find $\frac{d}{dx}\left(e^{-2006 \log_e x}\right)$.

24. Evaluate $\int \frac{ex}{\sqrt{2x^2 + 3}}\, dx$.

25. Find the area of the region bounded by the curve y = cosx, x - axis and the lines x = 0 and $x = \pi$.

26. Evaluate $\int \tan^2 x \sec^2 x \, dx$ **OR** Evaluate $\int \frac{1}{9 + 4x^2}\, dx$.

27. Evaluate $\int_1^{4013} (\operatorname{cosec}^{-1} x + \sec^{-1} x)\, dx$, $|x| \geq 1$.

28. Obtain the differential equation representing all the lines of family y = mx + c (where m and c are arbitrary constants).

29. If the distance of a particle executing rectilinear motion is x from a fixed point at time t, where $x = 2t^3 - 9t^2 + 12t + 8$, then when will the velocity become 0.

30. Two balls are thrown vertically upwards with velocities 19.6 m/s and 9.8 m/s. Find the height of the second ball, when the first ball attains maximum height.

SECTION C

- **Answer the following 31 to 40 questions as directed. Each question carries two marks. 20**

31. Prove by using slopes that A (12, 8) B (- 2, 6) and C (6, 0) are the vertices of a right triangle.

OR

Find the equation of the perpendicular bisector of $\overline{AB}$ where A is (- 3, 2) and B is (7, 6).

32. For the parabola $x^2 = 12y$, find the area of the triangle whose vertices are the vertex of the parabola and two end - points of its latus - rectum.

33. If the end - points of a chord of the ellipse $b^2x^2 + a^2y^2 - a^2b^2 = 0$ have eccentric angles with measure α and β, then prove that the equation of the line containing the chord is

$$\frac{x}{a}\cos\left(\frac{\alpha + \beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha - \beta}{2}\right).$$

34. If the eccentricities of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1$ are e_1 and e_2 respectively, then prove that $e_1^2 + e_2^2 = e_1^2 . e_2^2$.

OR

If the chord of hyperbola joining P (α) and Q (β) on the hyperbola subtends a right angle at the centre C (0, 0), then prove that $a^2 + b^2 \sin\alpha \sin\beta = 0$.

35. Prove that : $[\bar{x} + \bar{y} \quad \bar{y} + \bar{z} \quad \bar{z} + \bar{x}] = 2[\bar{x} \quad \bar{y} \quad \bar{z}]$.

36. If $\bar{x}, \bar{y}, \bar{z}$ are coplanar vectors, then prove that $\bar{x} + \bar{y}$, $\bar{y} + \bar{z}$, and $\bar{z} + \bar{x}$ are coplanar.

OR

If $(\bar{x} + \bar{y}).(\bar{x} - \bar{y}) = 63$ and $|\bar{x}| = 8|\bar{y}|$, then find $|\bar{x}|$.

37. Get the radius of the circle that is the intersection of the sphere $x^2 + y^2 + z^2 = 49$ and the plane $2x + 3y - z = 5\sqrt{14}$.

38. If $x = a(1 - \cos\theta)$, $y = a(\theta - \sin\theta)$, $\theta \in [0, \pi]$, $a \neq 0$, then find $\frac{d^2y}{dx^2}$.

39. Verify Rolle's theorem for $f(x) = \sin x + \cos x - 1,\ x \in \left[0, \frac{\pi}{2}\right]$. If it is applicable, find c.

OR

In which interval the function $f(x) = 5x^3 - 15x^2 - 120x + 3$ is increasing and in which interval is it decreasing ?

40. Evaluate $\int \frac{\sin x}{1 + \sin x}\,dx$.

S E C T I O N D

- **Answer the following 41 to 50 questions as directed. Each question carries 3 marks.** **30**

41. A is $(2\sqrt{2}, 0)$ and B is $(-2\sqrt{2}, 0)$. If $|AP - PB| = 4$, then find the equation of locus of P.

OR

Origin is circumcentre of triangle with vertices
$A(x_1, x_1 \tan\theta_1)$, $B(x_2, x_2 \tan\theta_2)$ and $C(x_3, x_3 \tan\theta_3)$ $(0 < \theta_i < \pi/2,\ x_i > 0,\ i = 1, 2, 3)$.
If the centroid of ΔABC is (x, y), prove that $\frac{y}{x} = \frac{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}{\cos\theta_1 + \cos\theta_2 + \cos\theta_3}$.

42. If the equation $3x^2 + (3 - p)xy + qy^2 - 2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

43. Forces measuring 5, 3 and 1 unit act in the direction (6, 2, 3), (3, -2, 6), (2, -3, -6) respectively. As a result, the particle moves from (2, -1, -3) to (5, -1, 1). Find the resultant force and work done.

44. Find the vector and Cartesian equations of the line passing through (1, 2, 3) and perpendicular to the two lines
$\bar{r} = (0, 0, 0) + K(1, 2, -1),\ K \in R$ and $\frac{x - 1}{3} = \frac{y}{2} = \frac{z}{6}$.

OR

Find the measure of the angle between two lines, if their direction cosines l, m, n satisfy $l + m + n = 0,\ l^2 + m^2 - n^2 = 0$.

45. Find the vector and Cartesian equations of the plane containing the lines
$\bar{r} = (1, 2, 3) + K(2, 3, 4),\ K \in R$ and $\frac{x - 1}{1} = \frac{y}{3} = \frac{z - 5}{4}$.

46. Find $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \frac{x - \cos(\sin^{-1}x)}{1 - \tan(\sin^{-1}x)}$.

47. Prove that if $x > 0$, then $\dfrac{x}{1+x^2} < \tan^{-1}x < x$.

48. Obtain $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\,dx$ as the limit of a sum.

49. Prove that $\displaystyle\int_8^{27} \frac{dx}{x - x^{\frac{1}{3}}} = \frac{3}{2}\log\left(\frac{8}{3}\right)$.

50. Solve $xy\dfrac{dy}{dx} = y + 2$. If $y(2) = 0$, then find the particular solution of the given differential equation.

OR

The population of a city increases at the rate of 3 % per year. How many years will it take for the population to double ?

SECTION E

- **Answer the following 51 to 54 questions. Each question carries 5 marks. 20**

51. A is $(-4, -5)$ in ΔABC and the lines $5x + 3y - 4 = 0$ and $3x + 8y + 13 = 0$ contain two of the altitudes of the triangle. Find the coordinates of B and C.

52. If $f(x) = \dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}$, $x \neq 0$, $f(0) = 1$, then prove that f is not continuous at $x = 0$.

OR

Find $\displaystyle\lim_{x \to 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$, $m, n \in N$.

53. If $x = \sin t$ and $y = \sin pt$, then prove that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + p^2y = 0$.

54. Evaluate $\displaystyle\int \frac{1}{1 + 5e^x + 6e^{2x}}\,dx$ **OR** Evaluate $\displaystyle\int \frac{\sec x}{1 + \operatorname{cosec} x}\,dx$.