Instructions :

1. **Answer all questions.**
2. **Write your answers according to the instructions given below with the questions.**
3. **Begin each section on a new page.**

SECTION - A

- **Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question.**

1. **If the co-ordinates of a point (6, -1) changes to (8, - 4), then write the co-ordinates of the point where the origin is shifted.**
   (a) (2, - 3) (b) (-3, 2) (c) (3, - 2) (d) (- 2, 3)

2. **Write the measure of the angle between the lines $x + y = 0$ and $y = [\pi]$**
   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) 0

3. **How many tangents to the circle $x^2 + y^2 = 29$ pass through the point (5, 2) ?**
   (a) 0 (b) 1 (c) 2 (d) none of these

4. **The standard equation of the parabola having vertex at the origin, passing through (-1, 1) and symmetric about y - axis is**
   (a) $y^2 = -x$ (b) $x^2 = y$ (c) $y^2 = x$ (d) $x^2 = -y$

5. **Write the equation of the auxiliary circle of $\frac{x^2}{4} - \frac{y^2}{9} = 1$.**
   (a) $x^2 + y^2 = -5$ (b) $x^2 + y^2 = 5$ (c) $x^2 + y^2 = 4$ (d) $x^2 + y^2 = 9$

6. **Obtain $\begin{vmatrix} \cos\theta\cos\alpha, & \cos\theta\sin\alpha, & \sin\theta \end{vmatrix}$**
   (a) - 1 (b) 0 (c) 1 (d) none of these

7. **Find the resultant force of (1, 2, - 1) and (1, - 2, 1)**
   (a) (2, 0, 0) (b) (- 1, 4, 2) (c) (2, 4, 2) (d) (- 2, 0, 0)

8. **Find the centre of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0$.**
   (a) (- 1, - 2, - 3) (b) (3, 2, 1) (c) (1, 2, 3) (d) (1, 2, - 3)

9. Find $\lim_{x \to 0} \frac{e^{3x} - 1}{x}$

(a) 3 (b) $\frac{1}{3}$ (c) log e^e (d) log e^3

10. Find $\frac{d}{dx}(\sin^2 x)$

(a) - sin 2x (b) $\cos^2 x$ (c) cos 2x (d) sin 2x

11. Find c, of Mean - value theorem for f(x) = log x, $x \in [1, e]$

(a) e - 1 (b) 1 - e (c) $1 - \frac{1}{e}$ (d) $\frac{1}{e - 1}$

12. Evaluate $\int \sin^2 (2x + 3)\, dx$

(a) $\frac{x}{2} - \frac{1}{8}\sin(4x + 6) + c$ (b) $\frac{x}{2} + \frac{1}{8}\sin(4x + 6) + c$

(c) $\frac{x}{2} - \frac{1}{4}\sin(2x + 3) + c$ (d) none of these

13. Obtain $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$

(a) - 2 (b) 2 (c) 0 (d) 1

14. Write the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + xy = 0$.

(a) 3 (b) 2 (c) 1 (d) none of these

15. A particle moves on a line and its distance from a fixed point at time t is x where $x = 4t^2 + 2t$, then at t = 2, find the acceleration.

(a) 2 (b) 4 (c) 6 (d) 8

SECTION B

- **Answer the following 16 to 30 questions. Each question carries one mark.**

15

16. Find the circumcentre of a triangle having vertices (0, 0), (3, 0) and (0, 4).

17. Find the length of the tangent from (6, - 5) to $x^2 + y^2 = 49$.

OR

If $12x + 5y + 16 = 0$ and $12x + 5y - 10 = 0$ are tangents of a circle, find the radius of the circle.

18. If $x + y + k = 0$ is a line containing the focal chord of the parabola $y^2 = 16x$, then find the value of k.

19. Find the measure of eccentric angle of the point $(2\sqrt{2}, 2)$ on the ellipse $\frac{x^2}{16} + \frac{y^2}{8} = 1$.

20. Find the direction cosines of $\vec{i} + \vec{j} + \vec{k}$.

21. If $\vec{a} = (1, - 1)$ and $\vec{b} = (1, 0)$, find $\cos \vec{a} \wedge \vec{b}$.

22. Find c, if the lines $\frac{x - 1}{c} = \frac{y + 2}{-2} = \frac{z - 3}{4}$ and $\frac{x - 5}{1} = \frac{y - 3}{1} = \frac{z + 1}{c}$ have the same direction.

23. Find the x – intercept of $2x - 3y + 6z = 12$.

24. Obtain $\frac{d}{dx}\left[\frac{3^x}{x^3}\right]$ **OR** Obtain $\frac{d}{dx}\left(5 \operatorname{cosec}^{-1} x\right)$.

25. Obtain $\int e^x \left[\frac{1 + x \log x}{x}\right] dx$ **OR** Obtain $\int \cos (\log x)\, dx$.

26. Obtain the area of the region bounded by y = x, X - axis, x = 0 and x = 3.

27. Find the value of $\int_{-1}^{1} \sin^3 x \cos^4 x \, dx$.

28. Obtain the differential equation of the family of curves $y = a \sin (x + b)$ (a and b are arbitrary constants).

29. A ball is projected vertically upwards with speed 19.6 m/s. Find its maximum height.

30. If $x = 2 - 3t + 4t^3$, then find the acceleration after 2 seconds.

SECTION C

- **Answer the following 31 to 40 questions as directed. Each question carries two marks. 20**

31. Find the equations of the lines through (2, 3) and making an angle of measure $\frac{2\pi}{3}$ with the Y - axis.

32. A quadrilateral ABCD is inscribed in a parabola. The sides $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{DA}$, make angles of measure $\theta_1, \theta_2, \theta_3$ and θ_4 with the axis of the parabola respectively. Prove that
$\cot\theta_1 + \cot\theta_3 = \cot\theta_2 + \cot\theta_4$.

33. If the length of the latus - rectum is 4 and distance between the foci is $4\sqrt{2}$, then find the standard equation of the ellipse.

OR

If the difference between the measures of the eccentric angles of P and Q is $\frac{\pi}{2}$ and if $\overleftrightarrow{PQ}$ cuts intercepts c and d on the axes, prove that $\frac{a^2}{c^2} + \frac{b^2}{d^2} = 2$.

34. If a line $lx + my + n = 0$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then show that $a^2l^2 - b^2m^2 = n^2$.

OR

If S and S' are the foci of a rectangular hyperbola with its centre C (0, 0), then prove that for any point P on the rectangular hyperbola
$SP \cdot S'P = CP^2$.

35. Prove that : $[\bar{x} + \bar{y} \quad \bar{y} + \bar{z} \quad \bar{z} + \bar{x}] = 2[\bar{x} \quad \bar{y} \quad \bar{z}]$.

36. Find a unit vector in R^3 making an angle of measure $\frac{\pi}{3}$ with each of the vectors (1, - 1, 0) and (0, 1, 1).

37. Find the centre and radius of the sphere $|\bar{r}|^2 - \bar{r}.(6, 12, 14) + 30 = 0$. Also express this equation in the Cartesian form.

38. If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$, then find y_2.

39. Find c, applying Mean - value theorem to $f(x) = \cos^{-1} x$, $x \in [-1, 0]$

OR

Divide 64 into two parts such that the sum of their cubes is minimum.

40. Obtain $\int \frac{x\,dx}{(1+x^2)(x^2-2)}$ OR Obtain $\int \frac{\log x - 1}{(\log x)^2} dx$.

SECTION D

- **Answer the following 41 to 50 questions as directed. Each question carries 3 marks. 30**

41. If A is (1, - 2) and B is (- 7, 1), find a point P on $\overleftrightarrow{AB}$ such that 3 AP = 5 AB.

42. Show that the circles $x^2 + y^2 + 6x + 2y - 90 = 0$ and $x^2 + y^2 - 34x - 28y + 260 = 0$ touch each other externally. Also find the equation of a line that contains the diameters of both the circles.

OR

If the circle $x^2 + y^2 + 2x + fy + k = 0$ touches both the axes, find f and k.

43. Use vectors to prove that for a ΔABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

OR

Prove that the angle in a semicircle is a right angle by vector method.

44. Find the measure of the angle between two lines, if their direction cosines (l, m, n) satisfy $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

45. Find the image of A (2, 3, 2) in the plane $\bar{r}$. (1, - 2, 1) = - 5.

OR

Find the foot of the perpendicular from (2, - 1, 2) to 2x - 3y + 4z = 44, the equation of this perpendicular and the perpendicular distance between the point and the plane.

46. Obtain $\lim_{x \to 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$, $m, n \in N$.

47. Find c, if Rolle's theorem is applicable for $f(x) = \sin^4 x + \cos^4 x$, $x \in \left[0, \frac{\pi}{2}\right]$.

48. Obtain $\int_0^1 \frac{\log(1+t)}{(1+t)^2} dt$.

49. If the line y = c, divides the area of the region bounded by the parabola $x^2 = 4y$ and the line y = 16 in two equal areas, find the value of c.

50. Solve : $\frac{dy}{dx} = \sin(x + y)$.

SECTION E

- **Answer the following 51 to 54 questions. Each question carries 5 marks. 20**

51. Find the equation of the line passing through the origin and containing a line - segment of length $\sqrt{10}$ between the lines 2x - y + 1 = 0 and 2x - y + 6 = 0.

OR

A (1, - 2) is a vertex of Δ ABC. Equations of the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$ are x - y + 5 = 0 and x + 2y = 0 respectively. Find the co-ordinates of B and C.

52. Prove that $\left\{\left(1 + \frac{1}{n}\right)^n\right\}$ is a bounded sequence (where $n \in N - \{1\}$).

53. If $y = x \cdot \log\left[\frac{x}{a + bx}\right]$, then prove that $x^3 y_2 = (xy_1 - y)^2$.

54. Obtain $\int \frac{x^2\, dx}{x^4 + 1}$ OR Obtain $\int \frac{2x + 3}{\sqrt{x^2 + x + 1}} dx$.