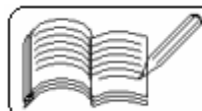


Instructions :

1. Answer all questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section on a new page.

SECTION - A

- Given below are 1 to 15 multiple choice questions. Each carries one mark. Write the serial number (a or b or c or d) in your answer book of the alternative which you feel is the correct answer of the question.
1. If the co-ordinates of a point (6, -1) changes to (8, -4), then write the co-ordinates of the point where the origin is shifted.
(a) (2, -3) (b) (-3, 2) (c) (3, -2) (d) (-2, 3)
 2. Write the measure of the angle between the lines $x + y = 0$ and $y = [\pi]$
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) 0
 3. How many tangents to the circle $x^2 + y^2 = 29$ pass through the point (5, 2)?
(a) 0 (b) 1 (c) 2 (d) none of these
 4. The standard equation of the parabola having vertex at the origin, passing through (-1, 1) and symmetric about y-axis is
(a) $y^2 = -x$ (b) $x^2 = y$ (c) $y^2 = x$ (d) $x^2 = -y$
 5. Write the equation of the auxiliary circle of $\frac{x^2}{4} - \frac{y^2}{9} = 1$.
(a) $x^2 + y^2 = -5$ (b) $x^2 + y^2 = 5$ (c) $x^2 + y^2 = 4$ (d) $x^2 + y^2 = 9$
 6. Obtain $|\cos\theta \cos\alpha, \cos\theta \sin\alpha, \sin\theta|$
(a) -1 (b) 0 (c) 1 (d) none of these
 7. Find the resultant force of (1, 2, -1) and (1, -2, 1)
(a) (2, 0, 0) (b) (-1, 4, 2) (c) (2, 4, 2) (d) (-2, 0, 0)
 8. Find the centre of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0$.
(a) (-1, -2, -3) (b) (3, 2, 1) (c) (1, 2, 3) (d) (1, 2, -3)



9. Find $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$

- (a) 3 (b) $\frac{1}{3}$ (c) $\log e^e$ (d) $\log e^3$

10. Find $\frac{d}{dx}(\sin^2 x)$

- (a) $-\sin 2x$ (b) $\cos^2 x$ (c) $\cos 2x$ (d) $\sin 2x$

11. Find c, of Mean - value theorem for $f(x) = \log x$, $x \in [1, e]$

- (a) $e - 1$ (b) $1 - e$ (c) $1 - \frac{1}{e}$ (d) $\frac{1}{e - 1}$

12. Evaluate $\int \sin^2(2x + 3) dx$

- (a) $\frac{x}{2} - \frac{1}{8} \sin(4x + 6) + c$ (b) $\frac{x}{2} + \frac{1}{8} \sin(4x + 6) + c$
 (c) $\frac{x}{2} - \frac{1}{4} \sin(2x + 3) + c$ (d) none of these

13. Obtain $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx$

- (a) -2 (b) 2 (c) 0 (d) 1

14. Write the degree of the differential equation $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + xy = 0$.

- (a) 3 (b) 2 (c) 1 (d) none of these

15. A particle moves on a line and its distance from a fixed point at time t is x where $x = 4t^2 + 2t$, then at $t = 2$, find the acceleration.

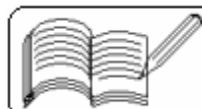
- (a) 2 (b) 4 (c) 6 (d) 8

SECTION B

- Answer the following 16 to 30 questions. Each question carries one mark.

15

16. Find the circumcentre of a triangle having vertices $(0, 0)$, $(3, 0)$ and $(0, 4)$.



17. Find the length of the tangent from $(6, -5)$ to $x^2 + y^2 = 49$.

OR

If $12x + 5y + 16 = 0$ and $12x + 5y - 10 = 0$ are tangents of a circle, find the radius of the circle.

18. If $x + y + k = 0$ is a line containing the focal chord of the parabola $y^2 = 16x$, then find the value of k .

19. Find the measure of eccentric angle of the point $(2\sqrt{2}, 2)$ on the ellipse $\frac{x^2}{16} + \frac{y^2}{8} = 1$.

20. Find the direction cosines of $\vec{i} + \vec{j} + \vec{k}$.

21. If $\vec{a} = (1, -1)$ and $\vec{b} = (1, 0)$, find $\cos \vec{a} \wedge \vec{b}$.

22. Find c , if the lines $\frac{x-1}{c} = \frac{y+2}{-2} = \frac{z-3}{4}$ and $\frac{x-5}{1} = \frac{y-3}{1} = \frac{z+1}{c}$ have the same direction.

23. Find the x -intercept of $2x - 3y + 6z = 12$.

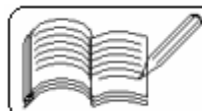
24. Obtain $\frac{d}{dx} \left[\frac{3^x}{x^3} \right]$ OR Obtain $\frac{d}{dx} (5 \operatorname{cosec}^{-1} x)$.

25. Obtain $\int e^x \left[\frac{1+x \log x}{x} \right] dx$ OR Obtain $\int \cos(\log x) dx$.

26. Obtain the area of the region bounded by $y = x$, X -axis, $x = 0$ and $x = 3$.

27. Find the value of $\int_{-1}^1 \sin^3 x \cos^4 x dx$.

28. Obtain the differential equation of the family of curves $y = a \sin(x+b)$ (a and b are arbitrary constants).



29. A ball is projected vertically upwards with speed 19.6 m/s. Find its maximum height.
30. If $x = 2 - 3t + 4t^3$, then find the acceleration after 2 seconds.

SECTION C

- Answer the following 31 to 40 questions as directed. Each question carries two marks. 20

31. Find the equations of the lines through (2, 3) and making an angle of measure $\frac{2\pi}{3}$ with the Y - axis.
32. A quadrilateral ABCD is inscribed in a parabola. The sides \overleftrightarrow{AB} , \overleftrightarrow{BC} , \overleftrightarrow{CD} and \overleftrightarrow{DA} , make angles of measure θ_1 , θ_2 , θ_3 and θ_4 with the axis of the parabola respectively. Prove that
 $\cot\theta_1 + \cot\theta_3 = \cot\theta_2 + \cot\theta_4$.
33. If the length of the latus - rectum is 4 and distance between the foci is $4\sqrt{2}$, then find the standard equation of the ellipse.

OR

If the difference between the measures of the eccentric angles of P and Q is $\frac{\pi}{2}$ and if

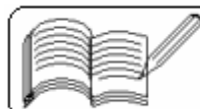
\overleftrightarrow{PQ} cuts intercepts c and d on the axes, prove that $\frac{a^2}{c^2} + \frac{b^2}{d^2} = 2$.

34. If a line $lx + my + n = 0$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then show that
 $a^2l^2 - b^2m^2 = n^2$.

OR

If S and S' are the foci of a rectangular hyperbola with its centre C (0, 0), then prove that for any point P on the rectangular hyperbola
 $SP \cdot S'P = CP^2$.

35. Prove that : $[\bar{x} + \bar{y} \quad \bar{y} + \bar{z} \quad \bar{z} + \bar{x}] = 2[\bar{x} \quad \bar{y} \quad \bar{z}]$.



36. Find a unit vector in \mathbb{R}^3 making an angle of measure $\frac{\pi}{3}$ with each of the vectors $(1, -1, 0)$ and $(0, 1, 1)$.
37. Find the centre and radius of the sphere $|\vec{r}|^2 - \vec{r} \cdot (6, 12, 14) + 30 = 0$. Also express this equation in the Cartesian form.
38. If $x = a(\cos\theta + \theta \sin\theta)$ and $y = a(\sin\theta - \theta \cos\theta)$, then find y_2 .
39. Find c , applying Mean - value theorem to $f(x) = \cos^{-1} x$, $x \in [-1, 0]$

OR

Divide 64 into two parts such that the sum of their cubes is minimum.

40. Obtain $\int \frac{x dx}{(1+x^2)(x^2-2)}$ OR Obtain $\int \frac{\log x - 1}{(\log x)^2} dx$.

SECTION D

- Answer the following 41 to 50 questions as directed. Each question carries 3 marks. 30

41. If A is $(1, -2)$ and B is $(-7, 1)$, find a point P on \overleftrightarrow{AB} such that $3AP = 5AB$.
42. Show that the circles $x^2 + y^2 + 6x + 2y - 90 = 0$ and $x^2 + y^2 - 34x - 28y + 260 = 0$ touch each other externally. Also find the equation of a line that contains the diameters of both the circles.

OR

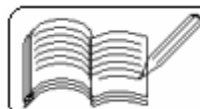
If the circle $x^2 + y^2 + 2x + fy + k = 0$ touches both the axes, find f and k .

43. Use vectors to prove that for a ΔABC , $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

OR

Prove that the angle in a semicircle is a right angle by vector method.

44. Find the measure of the angle between two lines, if their direction cosines (l, m, n) satisfy $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.



45. Find the image of A (2, 3, 2) in the plane $\bar{r} \cdot (1, -2, 1) = -5$.

OR

Find the foot of the perpendicular from (2, -1, 2) to $2x - 3y + 4z = 44$, the equation of this perpendicular and the perpendicular distance between the point and the plane.

46. Obtain $\lim_{x \rightarrow 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$, $m, n \in \mathbb{N}$.

47. Find c, if Rolle's theorem is applicable for $f(x) = \sin^4 x + \cos^4 x$, $x \in \left[0, \frac{\pi}{2}\right]$.

48. Obtain $\int_0^1 \frac{\log(1+t)}{(1+t)^2} dt$.

49. If the line $y = c$, divides the area of the region bounded by the parabola $x^2 = 4y$ and the line $y = 16$ in two equal areas, find the value of c.

50. Solve : $\frac{dy}{dx} = \sin(x + y)$.

SECTION E

• Answer the following 51 to 54 questions. Each question carries 5 marks. 20

51. Find the equation of the line passing through the origin and containing a line - segment of length $\sqrt{10}$ between the lines $2x - y + 1 = 0$ and $2x - y + 6 = 0$.

OR

A (1, -2) is a vertex of $\triangle ABC$. Equations of the perpendicular bisectors of \overline{AB} and \overline{AC} are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. Find the co-ordinates of B and C.

52. Prove that $\left\{ \left(1 + \frac{1}{n}\right)^n \right\}$ is a bounded sequence (where $n \in \mathbb{N} - \{1\}$).

53. If $y = x \cdot \log \left[\frac{x}{a + bx} \right]$, then prove that $x^3 y_2 = (xy_1 - y)^2$.

54. Obtain $\int \frac{x^2 dx}{x^4 + 1}$ OR Obtain $\int \frac{2x + 3}{\sqrt{x^2 + x + 1}} dx$.

