14.1 Nucleus - General Information

- A nucleus consists of electrically neutral neutrons and positively charged protons. The nucleus of hydrogen has only one proton and no neutrons.

- The charge of proton = the charge of electron = 1.6×10^{-19} C.

- Mass of proton, $m_p = 1.673 \times 10^{-27}$ kg,
  Mass of neutron, $m_n = 1.675 \times 10^{-27}$ kg.

- Protons and neutrons are commonly known as nucleons.

- The nucleus of any element is symbolically represented by $_ZX^A$ or $^A_Z X$.
  where, X is the chemical symbol of the element.
  Z is the atomic number of the element which represents the number of protons in its nucleus and shows the position of the element in the periodic table. In a neutral atom, the number of electrons is also Z.
  A is called the mass number of the element which represents the number of nucleons (protons + neutrons) inside the nucleus.
  N = A - Z represents the number of neutrons.
  e.g., the nucleus of carbon is represented by $_6C^{12}$ and that of uranium by $_{92}U^{238}$.
  Thus, nucleus of carbon has 6 protons in 12 nucleons and 12 - 6 = 6 neutrons and the nucleus of uranium has 92 protons in 238 nucleons and 238 - 92 = 146 neutrons.

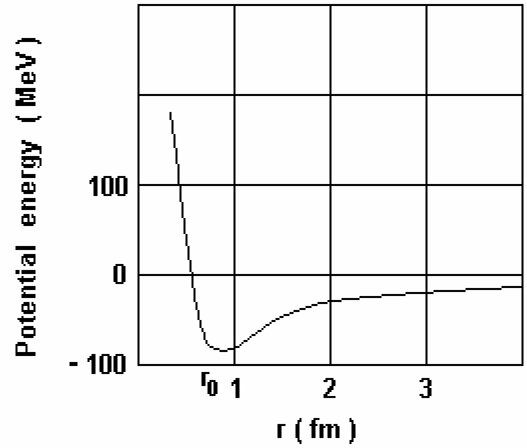
-

	same	different	examples
Isotopes	Z	A and N	Carbon - $_6C^{12}$, $_6C^{13}$ and $_6C^{14}$ Uranium - $_{92}U^{233}$, $_{92}U^{235}$ and $_{92}U^{238}$
Isobars	A	Z and N	Pb^{214} and Bi^{214}
Isotones	N = A - Z	A and Z	$_{36}Kr^{86}$, $_{37}Rb^{87}$
Isomers	A, Z, N (all)	different radioactive properties	Pair of isomers of $_{35}Br^{80}$

14.2 Nuclear Forces

Despite Coulombian force of repulsion between the protons in the nucleus, nucleus does not break up. This is because of strong nuclear force of attraction between (i) protons and protons, (ii) neutrons and neutrons and (iii) protons and neutrons which is more than the Coulombian force of repulsion. As far as this force is concerned, there is no difference between protons and neutrons. Hence they are commonly known as nucleons. This strong force is a short range force which exists between the neighbouring nucleons but is negligible between the nucleons far away from each other as in the large nuclei.

The figure shows the qualitative graph of their potential energy U (r) corresponding to the forces acting between two nucleons versus distance (r) between them.

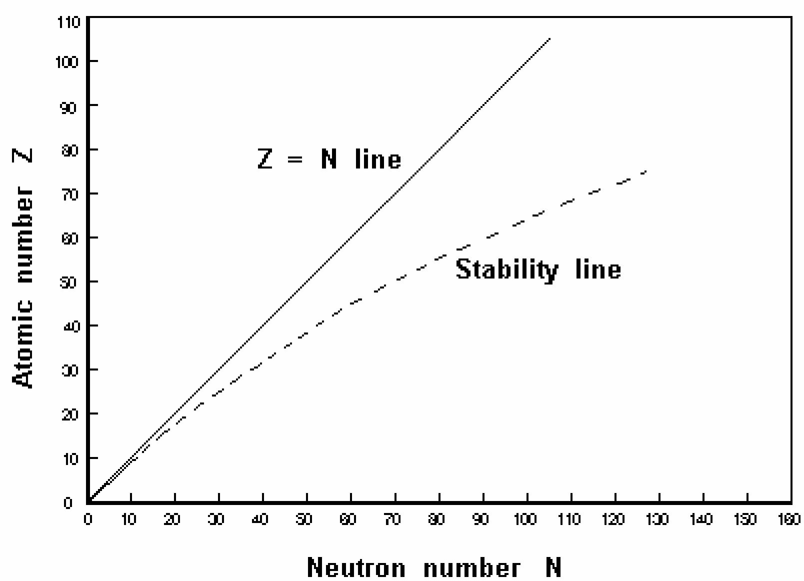


Here, $U(r) = -g^2 \dfrac{e^{-\frac{r}{R}}}{r}$ for $r > 1$ fm,

where R and g are constants and g is called the strength parameter. It can be seen from the graph that such forces act only unto 2 to 3 fm. For r < 1 fm, the forces are repulsive. This region of the nucleus is called its core.

14.3 Nuclear Stability

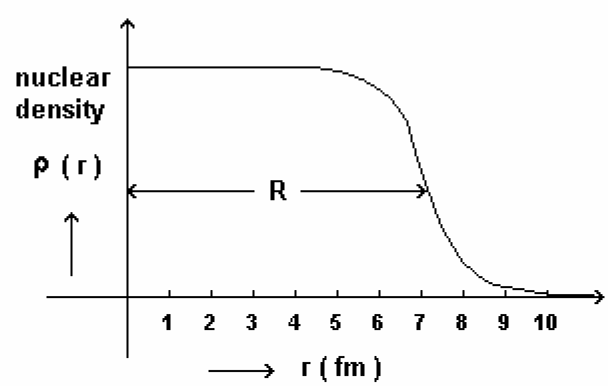
The figure shows graph of Z versus N, called Nuclide chart, for some stable nuclei.



In the nuclei of lighter elements, the number of protons (Z) and neutrons (N) are almost equal but in case of heavy elements, the number of neutrons is comparatively more. Stable nucleus lies on or very close to the stability line. Initially stability line is on Z = N line and then lies below it which is needed for the stability of the nucleus.

As the size of nucleus increases, the number of protons inside it also increase resulting in increase in the Coulombian repulsive force. To balance it, the nuclear force should also increase. Every additional proton exerts Coulombian repulsive force on all other protons inside the nucleus as this force is long ranged. But an additional neutron cannot exert the strong nuclear force on all the nucleons in a large nucleus as it is a short ranged force. Hence to balance the repulsive Coulombian force in a large nucleus, the number of neutrons have to be more than the number of protons.

14.4 Nuclear Radius



Rutherford estimated nuclear radius for the first time from experiments of scattering of α-particles on thin metal foils. Radius of nuclei of metals like gold, silver, copper was estimated to be about 10^{-14} m.

It can be seen from the graph of nuclear density ρ (r) versus the distance (r) that the nuclear density is uniform and maximum in the central region of nucleus and decreases gradually in the surface region. Though nucleus does not possess a sharp surface, its average radius can be given by the formula

$R = R_0 A^{1/3} = 1.1 A^{1/3}$ fm, where A is the atomic mass number of the nucleus,

e.g., the radius of nucleus of Au^{197} is $R \approx 1.1 \times (197)^{1/3}$ fm = 6.4 fm.

All the nuclei have almost same nuclear density as calculated from their mass and radii and is nearly 2×10^{17} kg m^{-3} which is 2×10^{14} times the density of water.

Unit of Mass and Energy in Atomic and Nuclear Physics:

In nuclear physics, the unit of mass is atomic mass unit denoted by u. "The twelfth part of the mass of an unexcited C^{12} atom is called 1 u."

1 u = 1.66×10^{-27} kg

In nuclear physics, the unit of energy is electron-volt denoted by eV. "The change in the energy of an electron passing through a potential difference of one volt is called 1 eV."

1 eV = 1.6×10^{-19} J;
KeV (kilo electron volt) = 10^3 eV; MeV (million electron volt) = 10^6 eV.

According to Einstein's theory of relativity, mass and energy are inter-convertible as given by

$E = mc^2$, where c is the velocity of light in vacuum. Using this,

1 u (mass) $\equiv$ 931.48 MeV (energy)

14.5 Binding Energy

Mass of any nucleus is less than the total mass of its nucleons in their free state.

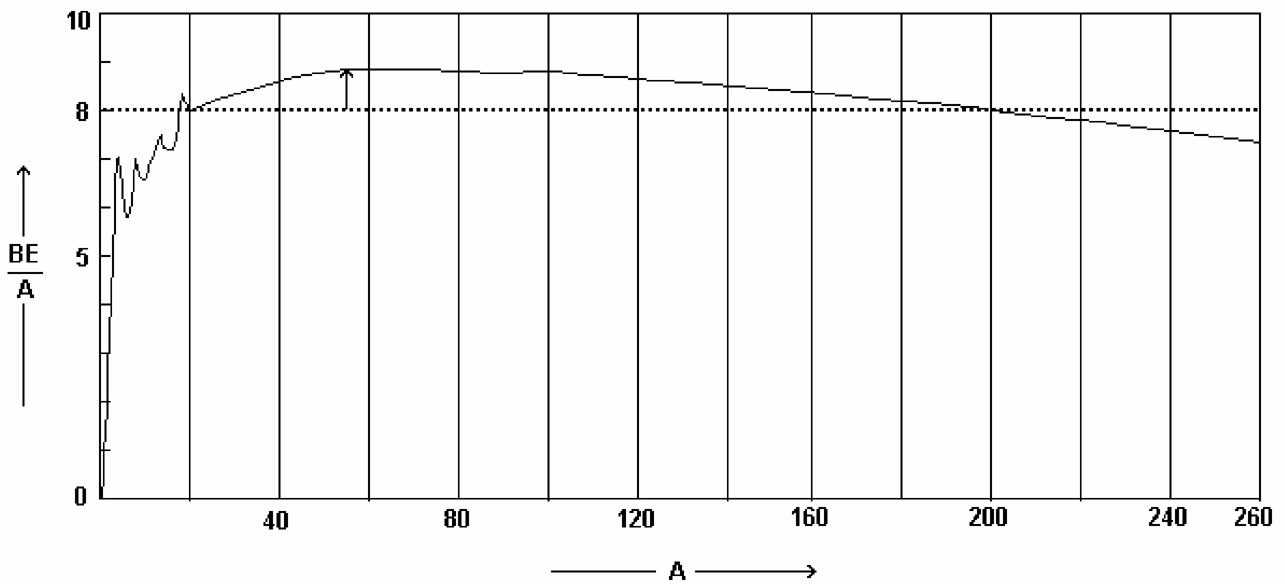
$\therefore$ $Z m_p + N m_n > M$, where M = mass of the nucleus,
Z and N = no. of protons and neutrons respectively in the nucleus
m_p and m_n = mass of one proton and one neutron respectively.

$(Z m_p + N m_n) - M = \Delta m$ is the mass defect and energy equivalent to it, $\Delta m c^2$, is called the "binding energy" (BE) of the nucleus.

For example, mass of deuteron ($_1H^2$) nucleus is 2.0141 u, whereas the total mass of a neutron and a proton is 2.0165 u.

$\therefore$ mass defect, Δm = 0.0024 u = 0.0024 $\times$ 931.48 = 2.24 MeV energy.

This is the binding energy of the deuteron nucleus. This much energy is needed to separate the neutron and the proton from the $_1H^2$ nucleus. Conversely, when a proton and a neutron are fused together to form $_1H^2$ nucleus, this much energy gets released. For deuteron nucleus, binding energy per nucleon is 2.24 / 2 = 1.12 MeV. Binding energy per nucleon is a measure of the stability of the nucleus.



The above figure shows the binding energy per nucleon versus atomic mass number A for different elements. The graph rises fast in the beginning, reaches a maximum at A = 56 for iron nucleus and then decreases slowly. For intermediate nuclei, the binding energy per nucleon is about 8 MeV and is independent of nuclear radius. For $Z \leq 10$ and for $Z \geq 70$, binding energy is small. Thus, intermediate nuclei are most stable as more energy is needed to free nucleus from them.

When a heavy nucleus is broken into two or more parts, energy is released. This process is called nuclear fission. Similarly, energy can be released by fusion of lighter nuclei also. This process is called nuclear fusion.

14.6 Natural Radioactivity

Heavy elements like uranium are unstable and emit invisible radiations spontaneously to gain stability. This phenomenon is called “natural radioactivity”. This was accidentally discovered by Becquerel in 1896 while studying the relation between X-rays discovered by Rontgen in 1895 and the phenomenon of fluorescence. He called them Becquerel rays.

Madame Curie and her husband Pierre Curie isolated radium and polonium from an ore of uranium called pitch-blend which showed much larger radioactivity than uranium. Later thorium and actinium possessing radioactivity were also discovered. Radiations from radioactive elements are called radioactive radiations.

Radioactive radiations are spontaneous and instantaneous and are not affected by pressure, temperature, electric and magnetic fields etc. Their emission rates also cannot be changed by any means not even by combining radioactive elements chemically with other elements to form different compounds.

14.7 Radioactive Radiations

During radioactive radiations, α-particles, β-particles and γ-rays are emitted. α-particles are material particles which are nuclei of helium having two protons and two neutrons and carry a charge of +2e. β-particles are electrons. The velocity of emission of α- and β-particles depends upon the radioactive element from which the emission occurs. γ-rays are not material particles but are electromagnetic rays.

All these radiations affect a photographic plate, produce fluorescence and ionize the medium through which they pass. The extent unto which they can penetrate a medium depends upon their energy and their interactions with the constituent particles of the medium. Their relative ionization and penetration power are as shown in the following table.

	α	β	γ
Relative Ionization Power	10,000	100	1
Relative Penetration Power	1	100	10,000

14.8 Radioactive decay constant

If ΔN out of N undecayed nuclei in a sample of a radioactive element decay in time ΔT, then

$\lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t} = \frac{dN}{dt}$ is called the rate of decay (or its activity, I) of the element at time t.

The rate of decay (or disintegration) of any element is proportional to the number of undecayed nuclei present at that time.

$\therefore \frac{dN}{dt} \propto -N$ (Negative sign indicates that N decreases with time.)

$$\therefore I = \frac{dN}{dt} = -\lambda N$$

λ is called the "radioactive constant" or the "decay constant" of that element. Its unit is s^{-1}. Each radioactive element and different unstable isotopes of the same element have different values of λ.

Large value of λ means higher decay rate and short life of the decaying element. Small value of λ means lower decay rate and long life of the decaying element. λ is not affected by temperature, pressure, etc.

14.9 Units of activity

"The activity of a sample is called 1 curie [1 Ci] if 3.7×10^{10} nuclear disintegrations occur per second." In practice, millicurie (1 mCi = 10^{-3} Ci) and microcurie (1 μCi = 10^{-6} Ci) are used more often.

"The activity of a substance is called 1 bequerel in which 1 disintegration occurs per second." It is denoted by 1 Bq.

14.10 Exponential Law of Radioactive Decay

For radioactive decay, $\frac{dN}{dt} = -\lambda N$ $\therefore$ $\frac{dN}{N} = -\lambda\, dt$

Integrating both sides,

$\ln N = -\lambda t + C$ $N = N_0$ at $t = 0$ gives $C = \ln N_0$

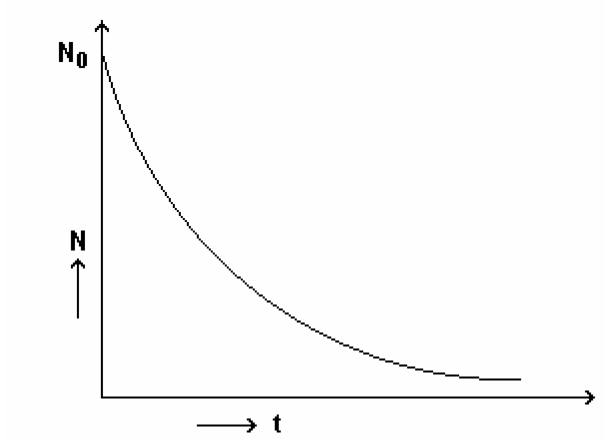
$\therefore$ $\ln N = -\lambda t + \ln N_0$

$\therefore$ $\ln \frac{N}{N_0} = -\lambda t$

$\therefore$ $N = N_0 e^{-\lambda t}$ and

$I = I_0 e^{-\lambda t}$

The above equations indicate that the number of undecayed nuclei and activity decrease exponentially with time. The graph of decay curve showing N or I versus t is shown in the figure.



14.11 Mean Lifetime

"The time interval in which the activity of a radioactive substance becomes eth part of its original activity is called its mean life τ."

Putting $I = I_0 / e$ at $t = \tau$ in the equation $I = I_0 e^{-\lambda t}$

$\frac{I_0}{e} = I_0 e^{-\lambda \tau}$ $\therefore$ $\frac{1}{e} = e^{-\lambda \tau}$

$\therefore$ $\tau = \frac{1}{\lambda}$

14.12 Half life

"The time interval, in which the number of nuclei of a radioactive element reduces to half of its number at the beginning of the interval, is called its half life ($\tau_{1/2}$)."

Putting $N = N_0 / 2$ at $t = \tau_{1/2}$ in the equation $N = N_0 e^{-\lambda t}$

$\frac{N_0}{2} = N_0 e^{-\lambda \tau_{1/2}}$ $\therefore$ $2 = e^{\lambda \tau_{1/2}}$

$\therefore$ $\ln 2 = \lambda \tau_{1/2}$

$\therefore$ $\tau_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

14.13 Emission of α - particles

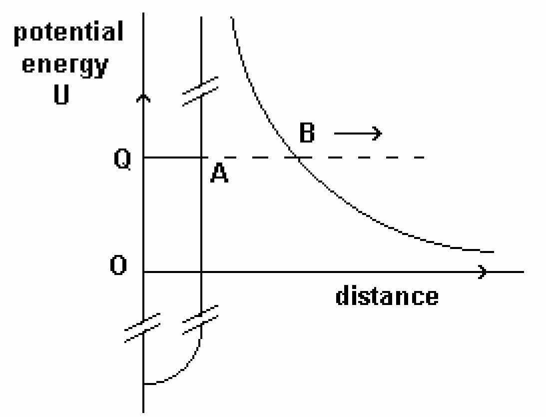
Most of the atoms having atomic number greater than Z = 83 emit α-particles which consist of two protons and two neutrons.

e.g., $_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4}$

The spontaneous emission of α-particle in the above case is possible because Q-value, which is energy equivalent of the mass defect, $\Delta m = M_U - (M_{Th} + M_{He})$, given by $\Delta m.c^2$ is positive.
At any time t, out of N undisintegrated nuclei, only dN = λNdt nuclei emit α-particles in a very small time interval dt. This is explained as under.

α-particle is formed inside the nucleus. In the case of above reaction, remaining part of the nucleus is $_{90}Th^{234}$. Now this α-particle moves freely inside the nucleus of $_{90}Th^{234}$ with Q amount of energy and experiences strong and attractive nuclear force and Coulombian repulsive force.

Thus the α-particle possesses some potential energy in the force field of the resultant of these two forces, variation of which with the distance is shown in the figure. When α-particle is at A, its potential energy is more than its maximum energy Q obtained by it due to the nuclear reaction above. This means its kinetic energy will be negative which is not possible according to classical physics and it also means that the α-particle will remain confined to the nucleus. But according to quantum mechanics, there is some probability of α-particle becoming free from the nucleus.



According to quantum mechanics, the probability of the α-particle breaking the barrier AB and going to the right side of B is

$$\approx \exp \; - \frac{4\pi}{h} \; a\sqrt{2m \; \langle U \rangle - E}$$

where, h = Planck's constant, a = width of the barrier (AB), < U > is the average height of the barrier and E = energy of the α-particle = Q.

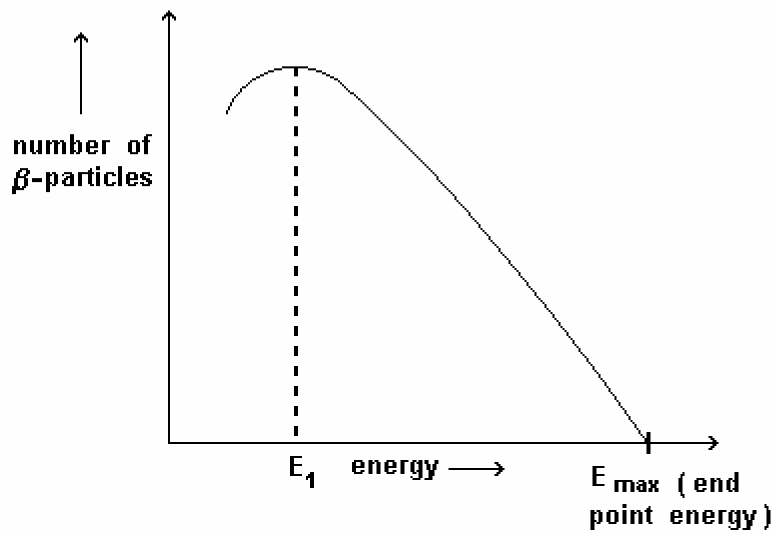
If there are N nuclei in a sample of $_{92}U^{238}$ and one α-particle is formed in each of them, then there will be N α-particles present. But only dN number of α-particles succeed in escaping the barrier in time dt. The phenomenon of penetration of particles having energy less than the height of the barrier (in terms of energy) is called tunneling.

Since the phenomenon of α-emission is probabilistic, any or all α-particles do not get emitted from the nucleus immediately and simultaneously after their formation.

14.14 β - emission

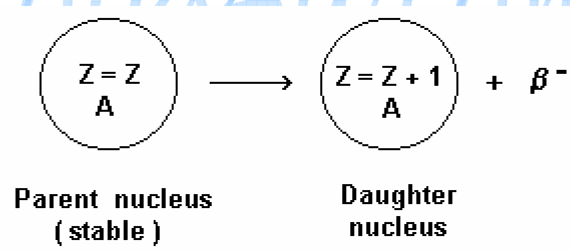
Just as β-particles (electrons) are emitted from the nucleus, positrons are also emitted from the nucleus. All its properties are identical to that of an electron except that its charge is positive. The electron ($_{-1}e^0$) can be called a negative β-particle (β^-) and a positron $_1e^0$ can be called a positive β-particle (β^+).

All the β-particles emitted from a radioactive element are not emitted with the same energy. The graph shows the number of β-particles emitted versus energy.



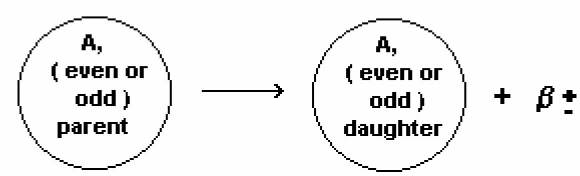
The number of β-particles having energy E_1 is maximum whereas E_{max} is the maximum energy of some β-particles. E_1 and E_{max} depend on the type of radioactive element.

Q amount of energy released when β-particle is emitted is distributed between the daughter nucleus and the β-particle. Emission of β-particle is shown in the figure.



Calculations indicate that a β-particle emitted from a nucleus must be emitted with a fixed value of energy whereas the graph shows that β-particles are emitted with different energies. Question arises as to what happens to the remaining energy of β-particles whose energy is less than E_{max} ?

It can be seen from the adjoining figure that the law of conservation of angular momentum is not satisfied for β-emission from nuclei having even as well as odd atomic mass number, A.



even $\frac{nh}{2\pi} \neq \frac{nh}{2\pi} + \frac{1}{2}\frac{h}{2\pi}$

odd $\left(n + \frac{1}{2}\right)\frac{h}{2\pi} \neq \left(n + \frac{1}{2}\right)\frac{h}{2\pi} + \frac{1}{2}\frac{h}{2\pi}$

A German scientist, Pauli, hypothesized that along with a β-particle, particles called neutrino or antineutrino having angular momentum $\frac{1}{2}\frac{h}{2\pi}$ is also emitted.

If the spins of neutrino and antineutrino are parallel, their total angular momentum will be $\frac{h}{2\pi}$ and the law of conservation of angular momentum holds for nuclei having even atomic mass number. If they have spins in the mutually opposite directions, the law of conservation of angular momentum holds for nuclei having odd atomic mass number.

When a proton is converted into a neutron inside the nucleus, a positron and a neutrino are emitted.

$p \rightarrow n + e^{+} + \nu$ (neutrino)

When a neutron is converted into a proton inside the nucleus, an electron and an antineutrino are emitted.

$n \rightarrow p + e^{-} + \bar{\nu}$ (antineutrino)

Z increases by 1 but A does not change when a β-particle is emitted.

14.15 γ - emission

Just as electromagnetic radiation is emitted during transition of atom from higher to lower energy level, radiation is emitted from the nucleus also. Atomic energy levels are of the order of eV whereas the nucleus energy levels are of the order of MeV. If the difference of energy between two layers equals 1 MeV, then

$$1 \text{ MeV} = 10^{6} \text{ eV} = 10^{6} \times 1.6 \times 10^{-19} \text{ J} = hf = hc/\lambda$$

$$\therefore \lambda = \frac{hc}{1.6 \times 10^{-13}} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^{8}}{1.6 \times 10^{-13}}$$

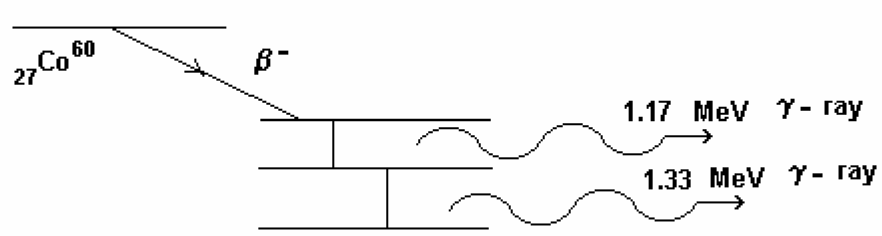
$$= 12.37 \times 10^{-13} \text{ m}$$

$$\approx 0.0012 \text{ nm}$$

The radiation having wavelength of this order is γ-radiation.

After the emission of an α-particle or a β-particle, the daughter nucleus is mostly found to be in the excited state. When such a nucleus experiences one or more transitions to the ground state, it emits γ-radiation.

For example as shown in the figure below, a ${}_{27}Co^{60}$ is in excited state after the emission of a β-particle. It moves to the ground state through two transitions and emits γ-rays of energy 1.17 MeV and 1.33 MeV.



14.16 Nuclear Reactions

In 1919 Rutherford discovered that one element can be converted into another by bombarding it with suitable particles with appropriate energy. This process is called artificial nuclear transmutation. Such a reaction is called nuclear reaction and is represented by

$$A + a \rightarrow B + b + Q,$$

where A is called the target nucleus, a the projectile, B the product and b the product particle. Q amount of energy liberated in the reaction is called Q-value of the process and is given by

$$Q = c^2 [M(A) + m(a) - M(B) - m(b)]$$

The process is called exoergic when $Q > 0$ and is called endoergic when $Q < 0$.

For example, Nitrogen can be transformed into Oxygen by bombarding α-particles on it.

$${}_7N^{14} + {}_2He^4 \rightarrow {}_8O^{17} + {}_1H^1 + Q$$

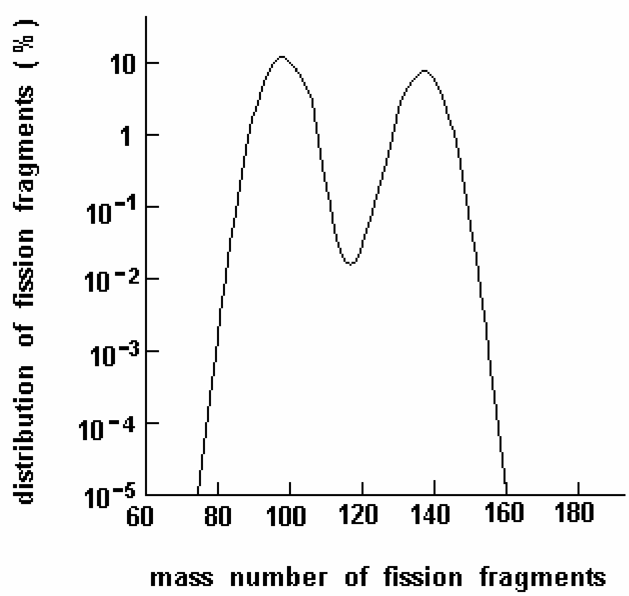
The laws of conservation of (1) momentum, (2) charge and (3) energy are obeyed in nuclear reactions. The sum of atomic numbers before and after the reaction are the same (the law of conservation of charge). Also, the sum of atomic mass numbers remain the same before and after the reaction.

14.17 Nuclear Fission

Neutrons are the best projectiles for nuclear fission because, being electrically neutral, they do not face Coulombian repulsive force while entering the nucleus.

Otto Hahn and Strassmann bombarded thermal neutrons (neutrons in thermal equilibrium with the surrounding at room temperature) on the solutions of compounds of uranium. In such experiments, they found new radioactive elements, one of which was Ba (Barium). This surprised them as to how an element of Z = 56 (Barium) could be formed from uranium of Z = 92 ! Lady physicist Meitner and her nephew Frisch suggested splitting of uranium into two almost equal fragments which process they called fission.

There are several theories explaining fission. To sum up, uranium nucleus absorbs a thermal neutron, gets into excited state and breaks up into two nearly equal fragments.
The phenomenon of fission is another case of tunneling. During fission, a barrier is formed including Coulombian forces and two fragments are formed by penetrating that barrier. In fact, not just two but 60 different fragments are formed as is clear from the figure.



The probability of formation of fragments of mass numbers A = 95 and A = 140 is maximum. One of such reactions can be given by the equation

$_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{51}Sb^{133} + {}_{41}Nb^{99} + 4\,{}_{0}n^{1}$

Four neutrons are also emitted in the above reaction. 200 MeV is released in the form of kinetic energy of γ-rays and energy of neutrons per fission of uranium nucleus.

14.18 Nuclear Chain Reaction and Nuclear Reactor

From the fission of one uranium nucleus by a single neutron, on an average, 2 to 3 neutrons are released which induce fission in more uranium nuclei. This sets up a self-sustaining chain of reaction called nuclear chain reaction. Following precautions have to be taken in carrying out such a reaction.

(1) The neutrons released during the fission are quite fast having kinetic energy of about 2 MeV. They have to be slowed down to the level of thermal neutrons (kinetic energy 0.04 MeV) so that they do not escape the fission material and induce further fission.

To slow down the neutrons, reflectors and moderators like heavy water (D_2O), carbon in the form of graphite, Beryllium and ordinary water are used in nuclear reactors. Moreover, the core region of the reactor is kept large to prevent the neutrons leakage from the surface.

(2) Large amount of energy released in nuclear reaction can raise the temperature to about 10^5 K. Hence to cool the fission material and moderators, coolants like suitable gases, water, liquid sodium, etc. are used.

(3) The ratio of number of neutrons produced to the number of neutrons incident at any stage of the chain reaction is called the multiplication factor (k). For controlled chain reaction, this ratio should be kept nearly one. With higher value of k, chain reaction can go out of control and with lower value of k, it may tend to stop. For this, some controlling rods of neutron absorbing materials like Boron and Cadmium are inserted in the fission material with automatic control device. Rods move further inside the fission material if k increases beyond one and come out if k reduces below one.

14.19 Nuclear reactor power plant

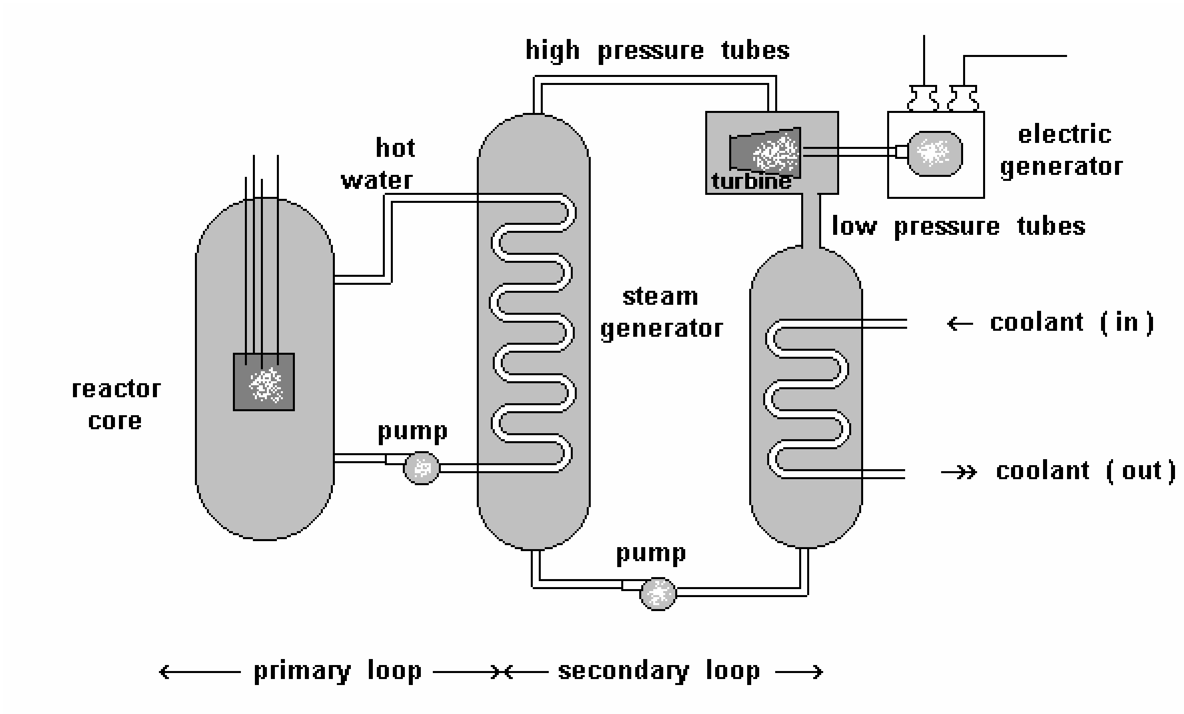
A schematic diagram of a specially designed nuclear power plant is shown in the figure on the next page. $_{92}U^{235}$ is used as fuel. But the ore of uranium contains 99.3 % of $_{92}U^{238}$ and only 0.7 % of $_{92}U^{235}$. Hence ore is enriched to contain 3 % of U^{235}. When $_{92}U^{238}$ absorbs neutron, it converts into plutonium though the following reactions:

$_{92}U^{238} + {}_{0}n^{1} \rightarrow {}_{92}U^{239} \rightarrow {}_{93}Np^{239} + e^{-} + \bar{\nu}$ (antineutrino)

$_{93}Np^{239} \rightarrow {}_{94}Pu^{239} + e^{-} + \bar{\nu}$ (antineutrino)

Plutonium is highly radioactive and can be fissioned with slow neutron.

In the reactor shown, ordinary water, used as a moderator and coolant, is circulated through the core of the reactor by means of a pump. Outlet water, at 150 atm and temperature of about 600 K is passed through a steam generator. The steam produced drives the turbine connected to electric generator and the low pressure outlet steam from the turbine is condensed, cooled and pumped back to the steam generator. Inside the reactor, some safety rods are used in addition to the controlling rods to quickly reduce the multiplication factor, k, below 1 in case of crisis.



In some specially designed reactors, uranium oxide pallets are used as fuel which are filled in long tubes from one end to another. The tubes are surrounded by the tubes through which liquid moderators are circulated. This device forms the core of the reactor.

14.20 Thermonuclear Fusion in Sun and Other Stars

Just as energy is released in nuclear fission, it is also released when light nuclei like proton and deuteron fuse together at a very high temperature to form helium nucleus. Such a process is called thermonuclear fusion.

Hydrogen works as fuel and helium is the end product of the process, called proton-proton cycle, occurring in the Sun. The reactions are given by

$${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + e^+ + \nu + 0.42 \text{ MeV} \qquad (1)$$

$$e^+ + e^- \rightarrow 2\gamma + 1.02 \text{ MeV} \qquad (2)$$

$${}_1H^2 \quad {}_1H^1 \rightarrow {}_2He^3 + \gamma + 5.49 \text{ MeV} \qquad (3)$$

$${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + {}_1H^1 + {}_1H^1 + 12.86 \text{ MeV} \qquad (4)$$

The first three reactions should occur twice so that the fourth reaction becomes possible. The total energy released in this process is $= 2 \times (0.42 + 1.02 + 5.49) + 12.86 = 26.7$ MeV.

Carbon-nitrogen cycle is also proposed in case of the Sun in which 25 MeV energy is released and the net reaction is fusion of 4 protons forming 1 nucleus of helium, ${}_2He^4$.