2.1 Line Integral of Electric Field

If a unit positive charge is displaced by $\vec{dl}$ in an electric field of intensity $\vec{E}$, work done is `given by

$$dW = \vec{E} \cdot \vec{dl}$$

Line integration of this equation gives the work done in displacing a unit positive charge from P to Q as

$$W = \int_P^Q \vec{E} \cdot \vec{dl}$$

This work depends only on the initial and final positions of the unit charge and not on the path followed by it. Hence, work done in moving a charge along a closed path is equal to zero. Thus electric field like gravitational field is a conservative field.

2.2 Electrostatic Potential

The work done by the electric field in moving a unit positive electric charge from an arbitrarily selected reference point θ, which may be inside or outside the field, to point P is given by

$$W_P = \int_\theta^P \vec{E} \cdot \vec{dl}$$

For the selected reference point, the value of W_P depends only on the position of point P and not on the path followed in going from reference point to point P.

Let θ be at infinity. The electric field at infinite distance due to finite charge distribution will be zero. The electric field due to an infinitely long charged plane at infinite distance will not be zero. However, in practice, one cannot have such a charge distribution.

The work done in a direction, opposing the electric field in bringing a unit positive charge from an infinite position to any point in the electric field is called the static electric potential (V) at that point.

Its sign is taken as negative as the work done is in a direction opposite to the electric field. Thus, work done in bringing a unit positive charge from infinity to points P and Q will be

$$V(P) = -\int_\infty^P \vec{E} \cdot \vec{dl} \quad \text{and} \quad V(Q) = -\int_\infty^Q \vec{E} \cdot \vec{dl}$$

$$\therefore \quad V(Q) - V(P) = -\int_\infty^Q \vec{E} \cdot \vec{dl} + \int_\infty^P \vec{E} \cdot \vec{dl} = \int_\infty^P \vec{E} \cdot \vec{dl} + \int_Q^\infty \vec{E} \cdot \vec{dl}$$

$$= -\int_P^Q \vec{E} \cdot \vec{dl}$$

This equation gives the electric potential of point Q with respect to point P. Its unit is volt (joule / coulomb) denoted by V and its dimensional formula is $M^1L^2T^{-3}A^{-1}$.

2.3 Electric Potential Energy and Potential Difference

A stationary electric charge at infinity has no energy (kinetic or potential) associated with it. If a unit positive charge is brought from infinity to an arbitrary point P in the electric field such that it has no velocity at that point then, the field being conservative, work done on it is stored with it in the form of potential energy and is called the electric potential of the point P and is given by

$$V(P) = -\int_{\infty}^{P} \vec{E} \cdot \vec{dl}$$

If the electric charge is of magnitude q instead of unity, then the work done is called the potential energy of the charge q at point P and is given by

$$U(P) = qV(P) = -\int_{\infty}^{P} q\vec{E} \cdot \vec{dl}$$

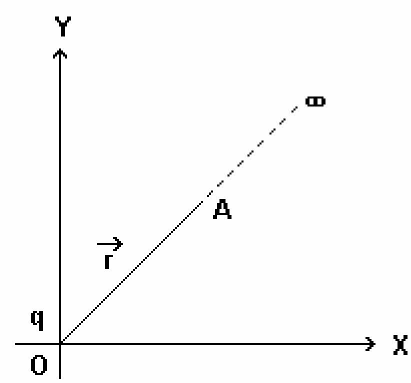
The original electric field or the arrangement of charges in the field should remain unaffected by bringing the electric charge q or the unit charge from an infinite distance to the point in the electric field.

Generally, one needs to calculate the potential difference or the difference in potential energy of charge q when it is moved from P to Q which is given by

$$U(Q) - U(P) = -\int_{P}^{Q} q\vec{E} \cdot \vec{dl}$$

It should be noted that this potential energy or the potential energy change is associated not only with the charge q but also with the entire charge distribution which gives rise to the electric field.

2.4 Electric Potential due to a Point Charge



The electric field at any point A having $\vec{r}$ as its position-vector due to a point charge q placed at the origin of the co-ordinate system is given by

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{kq}{r^2}\hat{r}$$

The electric potential at point A as shown in the figure is given by

$$V(A) = -\int_{\infty}^{A} \vec{E} \bullet \vec{dl}$$

Moving in the radial direction from infinity to point A, $\vec{dl} = dr\,\hat{r}$

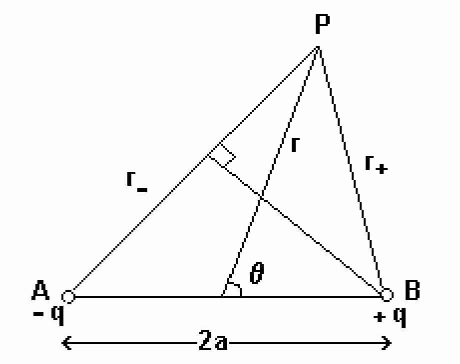
$$\therefore \quad V(A) = -\int_{\infty}^{A} \left(\frac{kq}{r^2}\hat{r}\right)\cdot(dr\,\hat{r}) = -kq\int_{\infty}^{A}\frac{dr}{r^2} = -kq\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{kq}{r}$$

$$\therefore \quad V(A) = V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

The electric potential is positive if q is positive and negative if q is negative. Hence, for negative charge negative sign should be used for q in the above equation.

2.5 Electric Potential in the Field of an Electric Dipole

Let the two charges -q and +q be placed at A and B respectively with a distance 2a between them.



Let P be a point at a distance r from the centre of this dipole and making an angle θ with its direction such that AP = r_- and BP = r_+.

The electric potential at P is given by

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_-}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r_- - r_+}{r_- r_+}\right]$$

If $r >> a$, then $r_+ \approx r_- \approx r$ and $r_- - r_+ \approx 2a\cos\theta$

As the atomic dipoles are of very small magnitude, this approximation holds good.

$$\therefore \quad V(r) = \frac{q}{4\pi\varepsilon_0}\left[\frac{2a\cos\theta}{r^2}\right] = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \qquad (\because\ p = 2aq)$$

As $p\cos\theta = \vec{p}\cdot\hat{r}$, $\quad V(\vec{r}) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\vec{p}\cdot\hat{r}}{r^2} \qquad (\text{for } |\vec{r}| >> 2a)$

Note: **(1) If $q \to \infty$ and $a \to 0$ in $p = 2qa$, then the dipole is called a point dipole.**

(2) The above equation gives the exact value of the electric potential for a point dipole and an approximate value for a dipole system other than a point dipole.

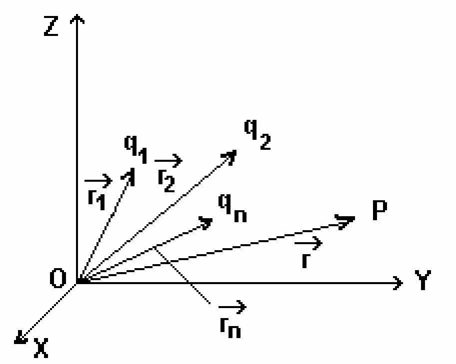
(3) For any point along the axis of the dipole, $\theta = 0$ or π and $V = \pm\dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^2}$.

(4) For a point along the equator of the dipole, $\theta = \pi/2$ and $V = 0$.

(5) In the case of a dipole, the electric potential varies as $1/r^2$ and not as $1/r$ as in the case of a point charge.

2.6 Electric Potentials due to Different Types of Charge Distributions

2.6 (a) Discrete Distribution of Charges:



The point electric charges $q_1, q_2, \dots q_n$ have position vectors $\vec{r}_1, \vec{r}_2, \dots \vec{r}_n$ respectively with respect to the origin.

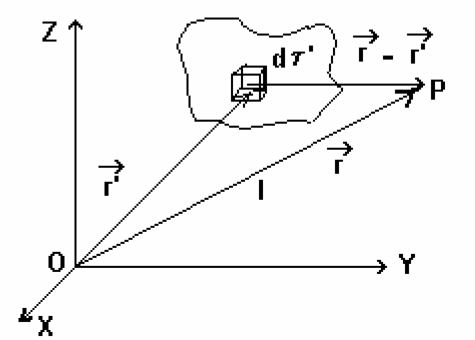
The total electric potential at point P having position vector $\vec{r}$ due to the entire charge distribution is equal to the algebraic sum of the electric potential due to each of the charges of the system and is given by

$$V = V_1 + V_2 + \dots + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{|\vec{r} - \vec{r}_1|} + \frac{q_2}{|\vec{r} - \vec{r}_2|} + \dots + \frac{q_n}{|\vec{r} - \vec{r}_n|}\right]$$

$$\therefore \; V(\vec{r}) = \sum_{i=1}^{n} \frac{kq_i}{|\vec{r} - \vec{r}_i|}$$

2.6 (b) Continuous Distribution of Charge:



Let $\rho(\vec{r}')$ = volume charge density for continuous charge distribution in some region.

$d\tau'$ = small volume element having position vector, $\vec{r}'$

The electric potential due to some volume element at some point P having position vector, $\vec{r}$, is given by

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\rho(\vec{r}')\,d\tau'}{|\vec{r} - \vec{r}'|}$$

Integrating, we get the electric potential at the point P due to entire charge of the system as

$$V(\vec{r}') = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}')\,d\tau'}{|\vec{r} - \vec{r}'|}$$

For constant charge distribution, $\rho(\vec{r}') = \rho$ can be taken as constant.

2.6 (c) Uniformly Charged Spherical Shell:

For a spherical shell of radius R carrying charge q on its surface, the electric potential at a point outside the shell at a distance r from its centre can be calculated by treating the entire charge of the shell as if concentrated at its centre and is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad (r \geq R)$$

The electric field inside the sphere is equal to zero. So the electric potential will be the same for all points inside the spherical shell and its value will be equal to the electric potential on the surface of the shell. Thus, the electric potential at the surface of the spherical shell and inside will be constant and is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad (r \leq R)$$

2.7 Equipotential Surfaces

If the electric potential at every point of any imaginary surface in an electric field is the same, then such a surface is called an equipotential surface.

As the electric potential at a distance r due to a point charge q is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r},$$

all concentric spherical surfaces with the charge q at the centre are equipotential surfaces.

The electric intensity vector at any point is always perpendicular to the equipotential surface passing through that point. This can be proved as under.

Work done for small displacement $\vec{dl}$ of a unit positive charge along the equipotential surface in a direction opposite to the electric field

$= - \vec{E} \cdot \vec{dl}$ = potential difference between the two points

= 0 (as the points are on equipotential surface)

$\therefore$ (E) (dl) cos θ = 0, where θ is the angle between $\vec{E}$ and $\vec{dl}$.

Since E $\neq$ 0 and dl $\neq$ 0, cos θ = 0

$\therefore$ cos θ = π / 2

which shows that $\vec{E}$ and $\vec{dl}$ are perpendicular to each other. As $\vec{dl}$ is tangential to the surface of the sphere, $\vec{E}$ is perpendicular to the spherical surface which is equipotential.
For uniform electric field, equipotential surfaces will be planes perpendicular to the electric field lines.

2.8 The Relation between Electric Field and Electric Potential

The electric potential difference between two close points P and Q is given by

$$dV = -\vec{E} \cdot \vec{dl}$$

If $\vec{E} = E_x \hat{i}$ **and** $\vec{dl} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$, **then**

$$dV = -(E_x \hat{i}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}) \quad \therefore \quad dV = -E_x\,dx \quad \text{and} \quad \frac{dV}{dx} = -E_x$$

Similarly, if the electric field exists along the Y and Z directions, then

$$\frac{dV}{dy} = -E_y \quad \text{and} \quad \frac{dV}{dz} = -E_z$$

If the electric field vector has all the three components (x, y, z), then the relation between the electric potential and the electric field can be given as

$$\frac{\partial V}{\partial x} = -E_x, \quad \frac{\partial V}{\partial y} = -E_y \quad \text{and} \quad \frac{\partial V}{\partial z} = -E_z \quad \text{or}$$

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Here $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$ **and** $\frac{\partial V}{\partial z}$ **are the partial derivatives of V (x, y, z) with respect to x, y and z respectively.**

While taking the partial derivative of V (x, y, z) with respect to x, the remaining variables y and z are taken constant. Such a derivative of V is called its partial derivative with respect to x and is denoted by $\frac{\partial V}{\partial x}$.

In general, if the electric field exists along the $\vec{r}$ **direction, then** $\frac{dV}{dr} = -E_r$.

The electric field is a conservative field and the above equations can only be used for a conservative field.

2.9 Potential Energy of a System of Charges

Work done in bringing electric charges from infinity to their respective positions in a system of charges gets stored in the form of electric potential energy of the system of charges.

The potential energy of a system of two charges q_1 **and** q_2 **is given as**

$$U_{12} = \frac{kq_1q_2}{r_{12}}$$

If a third charge q_3 is introduced in the system, then the potential energy of the system of three charges will be

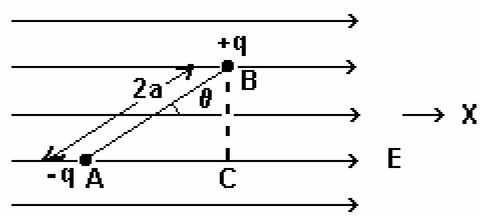
$$U = U_{12} + U_{13} + U_{23}$$

In general, if the system is made up of n electric charges,

$$U = \sum_{\substack{i=1 \\ i<j}}^{n} U_{ij} = \sum_{\substack{i=1 \\ i<j}}^{n} \frac{kq_iq_j}{r_{ij}}$$

2.9 (a) Potential Energy of an Electric Dipole in a Uniform Electric Field

The potential energy of an electric dipole AB in a uniform electric field ($\vec{E}$) as shown in the figure is equal to the algebraic sum of the potential energy of the two charges of the dipole.



Taking the electric potential near the negative charge to be zero, the potential energy of the dipole will be equal to the potential energy of the positive charge.

Let ΔV be the change in electric potential as we move from A to B.

$\therefore \quad \Delta V = - E \Delta x = - E (AC) = - E (2a \cos \theta)$

$\therefore$ the potential energy of electric charge q at point B is

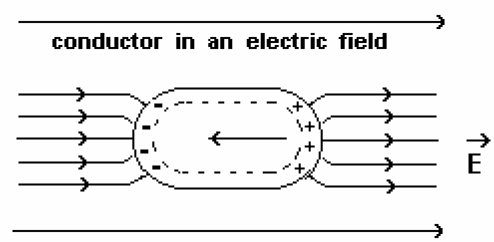
$$U = q \Delta V = - E (q.2a) \cos \theta = - E p \cos \theta = - \vec{E} \cdot \vec{p}$$

(i) When the electric dipole is parallel to the field, $- \vec{E} \cdot \vec{p} = - pE$ which means that the dipole has minimum potential energy (as it is more negative) which is also the equilibrium position of the dipole.

(ii) If $\theta = \pi$, the dipole has maximum potential energy, pE, and is in an unsteady position.

(iii) When the electric dipole is perpendicular to the field ($\theta = \pi / 2$), the potential energy of the dipole is zero.

2.10 Conductors and Electric Fields



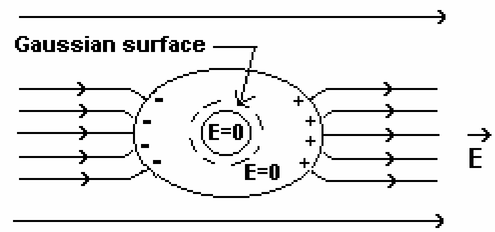
When a conducting material is placed in a uniform electric field as shown in the figure, free electrons migrate in a direction opposite to the electric field and get deposited on one side of the metal surface while the positive charge gets deposited on the other side of the conductor. This produces an electric field inside the conductor and the migration of charges stops when the internal electric field becomes equal to the external field.

If we draw a Gaussian surface inside the conductor as shown in the figure, then since the electric field on it is zero, the net electric charge enclosed by it is also zero.

The important conclusions are:

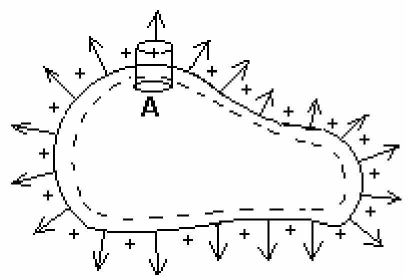
(1) Stationary electric charge distribution is induced on the surface of the conductor.
(2) Both the electric field and the net electric charge inside the conductor are zero.
(3) At every point on the outer surface of the conductor, the electric field is perpendicular to the surface. This is so because the electric charge on the surface is stationary which means that no tangential force acts on it, thus proving that the electric field on the surface has no tangential component.

Consider another example of a hollow conductor placed in an external electric field.



Here also, the electric charges deposit on the outer surfaces and the electric field inside is zero as there is no charge inside. This phenomenon is called Electro-static Shielding. When a car is struck by lightning, the person sitting inside is saved from lightning as the car is hollow and acts like an electrostatic shield.

Electric field inside a charged conductor which is NOT in an electric field is also zero. Consider a Gaussian surface close to the surface of the conductor as shown by broken line in the figure. The line integration of the electric field along the Gaussian surface being zero, the net electric charge enclosed by it is also zero. This shows that in a charged conductor, the electric charge gets distributed on the outer surface of the conductor.



As the electric charges are stationary, the direction of the electric field will be perpendicular to the surface of the conductor as shown in the figure and its magnitude will be $\dfrac{\sigma}{\varepsilon_0}$.

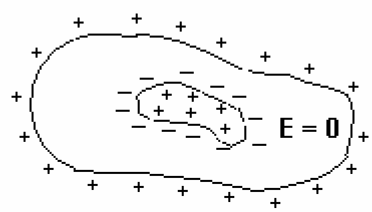
To explain it, consider a pillbox shaped Gaussian surface on the surface of the conductor as shown in the above figure.

The charge enclosed by the Gaussian surface = $A \cdot \sigma$
The total flux passing through this surface = $A E$

$\therefore$ by Gauss's law, $A E = A \dfrac{\sigma}{\varepsilon_0}$ $\quad \therefore \; E = \dfrac{\sigma}{\varepsilon_0}$

If σ is not uniform along the surface, its proper value at the point should be used to calculate value of E at that point.

If a positive electric charge is placed in the cavity of the conductor as shown in the adjoining figure, it induces charges on the inner and outer surfaces of the conductor in such a way that the field will be zero in the interior portion of the conductor.



2.11 Capacitors and Capacitance

When positive electric charge on an isolated conducting sphere shown in figure 1 is gradually increased, electric potential on its surface and electric field in its vicinity increase. When the electric field becomes strong, it ionizes the surrounding air which causes charge on the sphere to leak and the charge on the sphere cannot be increased further. During this process, the ratio of charge (Q) and the electric potential (V) of the sphere remains constant. This ratio, Q / V, is called its capacitance (C).

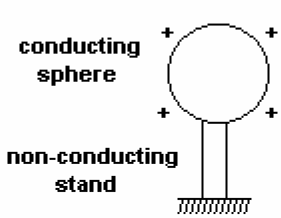


Fig. 1

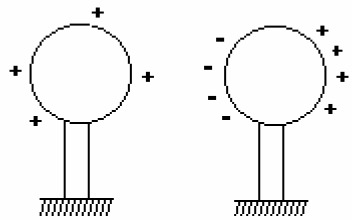


Fig. 2

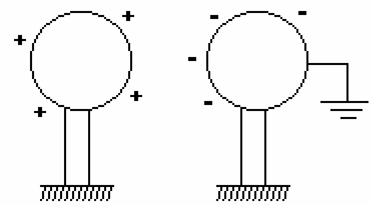


Fig. 3

To increase capacitance C of the sphere, another isolated conducting sphere is brought near it on which charge is induced as shown in figure 2. On earthing, the positive charge gets neutralized as shown in figure 3. The negative charge induced on the second sphere reduces the electric potential of the first sphere thereby increasing its charge storage capacity. The ratio Q / V of the charge on the first sphere and the potential difference between the two spheres is still constant and is called the capacitance C of the system. The value of C depends on the dimensions of the spheres, the distance between the two spheres and the medium between them.

The arrangement in which two good conductors of arbitrary shape and volume, are arranged close to one another, but separated from each other, is called a capacitor.

The conductors are known as plates of the capacitor. Positively charged conductor is called the positive plate and the negatively charged conductor the negative plate. Both the plates are equally charged. The charge on the positive plate is called the charge (Q) of the capacitor and taking the potential difference between the two plates as V, capacitance of the capacitor is C = Q / V.

The S.I. unit of capacitance is coulomb / volt which is also called farad (F) named after the great scientist Michael Faraday. The smaller units of farad are microfarad ($\mu F = 10^{-6}$ F) and picofarad (pF = 10^{-12} F).

2.12 Parallel Plate Capacitor

This type of capacitor is made by two metallic plates having identical area and kept parallel to each other. The distance (d) between the two plates is kept less as compared to the dimensions of the plates to minimize non-uniform electric field due to the irregular distribution of charges near the edges.

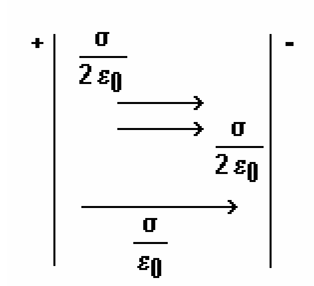
Let Q = electric charge on the capacitor

$\therefore \quad \sigma = Q / A$ = surface charge density

As d is very small, the plates can be considered as infinitely charged planes and the electric field between the plates can therefore be considered uniform.

The electric fields, E_1 and E_2, between the plates due to positively charged and negatively charged plates respectively are equal in magnitude and direction. The direction of both E_1 and E_2 is from the positively charged plate to the negatively charged plate.

The resultant electric field between the plates is, therefore,



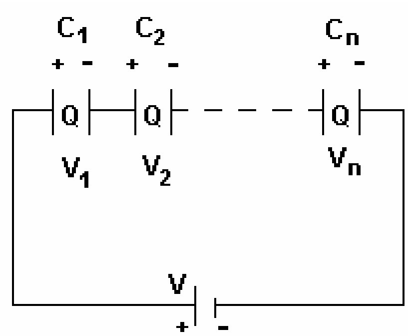
$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad (\because \sigma = \frac{Q}{A})$$

Outside the plates, E_1 and E_2 being oppositely directed cancel each other resulting in zero electric field in this region.

The potential difference between the two plates is

$$V = E\,d = \frac{Q}{\varepsilon_0 A} d \qquad \therefore \text{ the capacitance of the capacitor, } C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

2.13 (a) Series Connection of Capacitors



The end to end connection of capacitors as shown In the figure is called the series connection of capacitors.

Equal charge Q deposits on each capacitor, but the p.d. between their plates is different depending on the value of its capacitance.

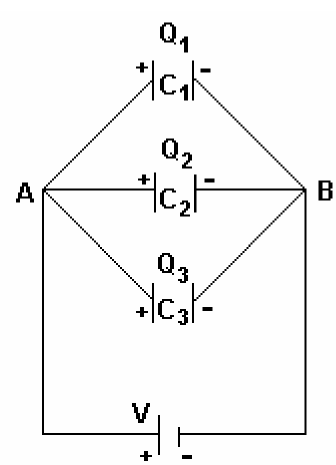
$$\therefore \quad V = V_1 + V_2 + \dots + V_n$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \dots + \frac{Q}{C_n}$$

$$\therefore \quad \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

When all capacitors connected in series are replaced by a single capacitor of capacitance C such that the charge deposited on it is Q with the same voltage supply, then such a capacitor is called their equivalent capacitor.

$$\therefore \quad \frac{V}{Q} = \frac{1}{C} \qquad \therefore \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$



The value of C is smaller than the smallest of C_1, C_2, ... C_n.

2.13 (b) Parallel Connection of Capacitors

The connection of capacitors in which positive plates of all capacitors are connected to a single point and negative plates to another single point in a circuit is called parallel connection of capacitors as shown in the figure. In such a connection, charge accumulated on each of the capacitors is different depending on the value of its capacitance, but the p.d. across all is the same.

Thus, total charge $Q = Q_1 + Q_2 + Q_3 + ...$
$= (C_1 + C_2 + C_3 + ...) V$

When all capacitors connected in parallel are replaced by a single capacitor of capacitance C such that the charge deposited on it is Q with the same voltage supply, then such a capacitor is called their equivalent capacitor. Its value is

$C = Q / V = C_1 + C_2 + C_3 + ...$

2.14 Energy Stored in a Charged Capacitor

The work done in charging the capacitor gets stored in it in the form of potential energy.

The electric field on one plate due to charge Q and charge density σ on it $= \dfrac{\sigma}{2\varepsilon_0}$

The potential energy of one plate at a distance d from the other plate

U_E = (electric p.d. between them) × (electric charge on the second plate)

$$= \frac{\sigma d}{2\varepsilon_0} \times Q$$ (taking reference potential on the first plate as zero).

Putting $\sigma = \dfrac{Q}{A}$,

$$U_E = \frac{Q}{A} \cdot \frac{d}{2\varepsilon_0} \cdot Q = \frac{Q^2}{2 \cdot \varepsilon_0 A / d} = \frac{Q^2}{2C}$$

Putting $Q = CV$ or $C = Q / V$, we get

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

<u>Energy Stored in a Capacitor in terms of Energy Density of Electric Field</u>

If A = area of the plates of the capacitor
d = separation between the plates

then energy density of electric field in the capacitor

ρ_E = energy stored per unit volume

$$= \frac{U_E}{A d} = \frac{1}{2} \frac{CV^2}{A d} = \frac{1}{2} \frac{\varepsilon_0 A}{d} \cdot \frac{V^2}{A d} = \frac{1}{2} \varepsilon_0 \left(\frac{V}{d}\right)^2$$

$$= \frac{1}{2} \varepsilon_0 E^2 \quad (\because \frac{V}{d} = E)$$

The above equation gives energy stored in a capacitor in terms of the energy stored in the electric field between the two plates. This is a general result which holds true for an electric field due to any charge distribution.

2.15 Dielectric Substances and their Polarization

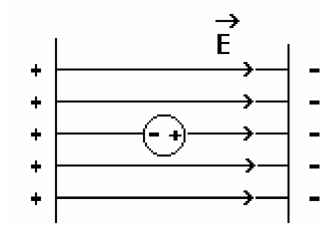
- A non-conducting material is called a dielectric.

- The dielectric does not possess free electrons like a conducting material.
- Introduction of a dielectric between the two parallel plates of a capacitor considerably increases the capacitance of a capacitor.
- Dielectric materials are of two types: (1) Polar and (2) Non-polar.
- The atoms of a dielectric (like HCl, H_2O) that have permanent dipole moment is called a polar dielectric.
- The atoms of a dielectric (like H_2, O_2) that do not have permanent dipole moment is called a non-polar dielectric.

2.15 (a) Non-polar Atoms (or Molecules) placed in a Uniform Electric Field:

The centres of the positive and the negative charges coincide in a non-polar dielectric atom or molecule. So it does not have permanent dipole moment.

If such an atom or molecule is placed in a uniform electric field ($\vec{E}$), the centres of the positive and negative electric charges get separated due to electric force acting on them in opposite directions. This is known as polarization of the atom.

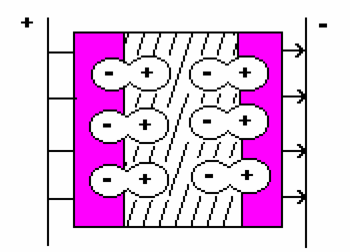


If the electric field is not very strong, then the dipole moment, $\vec{p}$, of the induced dipole is directly proportional to $\vec{E}$.

$\vec{p} = \alpha \vec{E}$, where constant α is called the atomic (or molecular) polarizability.

2.15 (b) Dielectric in a Capacitor

A dielectric slab of polar material is placed between the parallel plates of a capacitor as shown in the figure. The torque due to the electric field acting on atomic dipoles aligns them in the direction of the electric field. In a solid dielectric material, its atoms perform oscillatory motion, whereas in liquid or gas dielectric materials, they perform linear motion. The dipoles are in equilibrium and completely aligned as shown in the figure when its electric potential energy, $- \vec{p} \cdot \vec{E}$, equals the thermal energy, $\frac{3}{2} k T$, due to absolute temperature T.



It can be seen in the figure that the atomic dipoles line up such that the opposite polarity face each other canceling each other's effect. The net charge of the system is the charge of the capacitor plates. The dipole closer to the positive plate has negative charge lined up facing it and vice versa. These induced electrical charges are called bound charges.

The electric field between the capacitor plates is

$E_f = \dfrac{\sigma_f}{\varepsilon_0}$ (1) in the absence of the dielectric medium and is

$E = \dfrac{\sigma_f - \sigma_b}{\varepsilon_0}$ (2) in the presence of the dielectric slab, where σ_f is the free (controllable) surface charge density on the surface of the capacitor plates and σ_b is the bound surface charge density on the surface of the dielectric slab. σ_b is taken negative as the surface charges of the dielectric slab have opposite polarity as compared to the charges of the capacitor plates.

Determining σ_b

The dipole moment of the dielectric slab having surface area A and length L is

$$P_t = \sigma_b A L \quad \therefore \quad \frac{P_t}{A L} = \frac{P_t}{V} = \sigma_b = P,$$

where P_t / V is the dipole moment of the dielectric slab per unit volume and P is called the intensity of polarization.

If the electric field is not very strong, the ratio of P and the electric field is a constant.

$$\therefore \quad \chi_e = \frac{P}{E}$$

This ratio χ_e is called the electric susceptibility of the dielectric material and its value depends on the dielectric material and its temperature.

$$\therefore \quad P = \sigma_b = \chi_e E$$

$$\therefore \quad E = \frac{\sigma_f}{\varepsilon_0} - \frac{P}{\varepsilon_0} = \frac{\sigma_f}{\varepsilon_0} - \frac{\chi_e E}{\varepsilon_0} \quad \text{[using equations (1) and (2)]}$$

$$\therefore \quad E (\varepsilon_0 + \chi_e) = \sigma_f = \varepsilon_0 E_f$$

$$\therefore \quad E = \left(\frac{\varepsilon_0}{\varepsilon_0 + \chi_e} \right) E_f$$

Here, $\varepsilon_0 + \chi_e = \varepsilon$ is called the permittivity of the medium and its value depends on the type of the dielectric material and its temperature. Its unit is $C^2 N^{-1} m^{-2}$.

$$\therefore \quad E = \frac{\varepsilon_0}{\varepsilon} E_f = \frac{E_f}{\varepsilon / \varepsilon_0} = \frac{E_f}{\varepsilon_r} = \frac{E_f}{K}$$

$\dfrac{\varepsilon}{\varepsilon_0}$ is the relative permittivity ε_r and is also called dielectric constant K.

This shows that when a dielectric material having dielectric constant K is placed as a medium between the capacitor plates, the electric field is reduced by a factor of K and the capacitance of the capacitor becomes

$$C' = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 K A}{d} = K C$$

Thus when a dielectric medium having dielectric constant K is introduced in a capacitor, the value of its capacitance increases to K times the original capacitance.

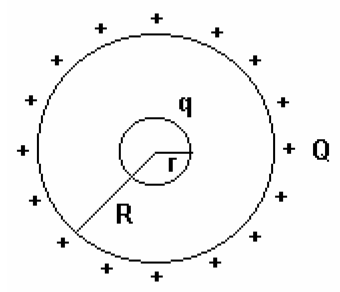
From equation (2), we have $\varepsilon_0 E + P = \sigma_f$ ($\because \sigma_b = P$)

Here, E and P are electric and polarization fields respectively which are both vector quantities.

$\therefore$ we can write $\varepsilon_0 \vec{E} + \vec{P} = \vec{D}$, where $\vec{D}$ is called the displacement field.

2.16 Van-de-Graf Generator

The principle of this machine is as under.



A conducting sphere having charge q and radius r is kept centrally inside a conducting shell having charge Q and radius R (r < R).

The electric potential on the spherical shell of radius R and on the surface of the sphere of radius r are respectively

$$V_R = \frac{kQ}{R} + \frac{kq}{R} \quad \text{and} \quad V_r = \frac{kQ}{R} + \frac{kq}{r}$$

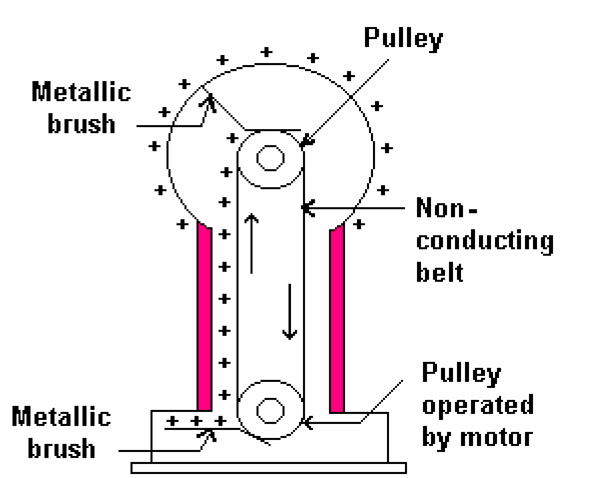
Hence, the potential difference between the surfaces of the two spheres is

$$V_r - V_R = \frac{kQ}{R} + \frac{kq}{r} - \frac{kQ}{R} - \frac{kq}{R} = kq\left(\frac{1}{r} - \frac{1}{R}\right)$$

The above equation shows that the smaller sphere is at a higher potential as compared to the large spherical shell. So if the two are connected through a conductor, then the electric charge will flow from the smaller sphere to the larger spherical shell. So if charge is transferred continuously to the smaller sphere in some way, then it will accumulate on the larger shell thereby increasing its electric potential to a very large value.

Van-de-Graf used this principle to construct a generator known as Van-de-Graf generator to generate several million volts of electric potential.

As shown in the figure, a non-conducting belt is driven across two pulleys. The lower pulley is connected to a motor and the upper one is surrounded by a spherical shell. Positive electric charge is transferred to the belt near the lower pulley using a discharge tube and a brush with sharp edges. The electric charge moves to the upper pulley and is deposited on the outer shell with a metallic brush. Thus electric potential of the order of 6 to 8 million volts is generated on the outer shell.



The highly intense electric field produced in this device can accelerate electric charges which can be used to study the composition of matter at the microscopic level.