14 - OSCILLATIONS

1) An oscillator having a mass of 0.2 kg is executing simple harmonic oscillations. After 100 oscillations the amplitude becomes $1 / 7^{th}$ of the original value. Find damping coefficient of this motion, if its periodic time is 2 seconds. [March, 2003]
(Ans: 3.89 dyne-sec /cm)

2) An oscillator having a mass of 100 gm is executing damped oscillations. After 1500 oscillations, the amplitude reduces to half of its original value. Find the damping coefficient if its time period is 2.22 seconds. [March, 2001]
(Ans: 0.042 dyne-sec /cm)

3) For damped oscillations, find the time for decrease of the amplitude to A / 32. Mass of oscillator = 200 gm and resistive force coefficient = 0.1 dyne-sec /cm [March, 2001]
(Ans: 3.85 hour)

4) In a simple harmonic motion, the velocities of a particle are 10 cm /s and 24 cm /s when its displacements are 12 cm and 5 cm respectively. Calculate its periodic time and amplitude. [October, 1998]
(Ans: 3.14 sec, 13 cm)

5) Find the resistive force on a body when its velocity is 7200 km/hr in a medium of damping constant 10^{-3} N-s /m. [March, 1998]
(Ans: 2 N)

6) A body of mass 'm' is suspended at the end of a spring. The periodic time of the body is 4 seconds. If the mass is increased by 4 kg, the increase in periodic time is found to be 2 seconds. Calculate the mass 'm'. [October, 1997; October, 1991]
(Ans: 3.2 kg)

7) Angular frequency of S. H. oscillator is 2 rad /s. Initial displacement is 5 cm and initial velocity is 10 cm /s. Give equation for this S. H. M. [March, 1997]
[Ans : $5\sqrt{2}\ \sin(2t + \pi / 4)$ cm]

8) Find frequency of necessary external oscillating force to produce resonance in the seconds pendulum. [March, 1996]
(Ans: 0.5 Hz)

9) For a S. H. O., mass, length of path of oscillation, frequency and initial phase are 0.5 kg, 10 cm, 60 oscillations /minute and $\pi / 3$ radian respectively. Write an equation of displacement at time t. Find its force constant and mechanical energy. [October, 1995]
(Ans: $y = 0.1 \sin(2\pi t + \pi / 3)$ m, 19.72 N /m, 0.025 joule)

10) A particle executes S. H. M. on a line 4 cm long. Its velocity when passing through the centre of the line is 12 cm /s. Find the period. [March, 1995]
(Ans: 1.05 seconds)

11) When and where will the potential and kinetic energy of an oscillator become equal starting its motion from mean position along Y-axis and having periodic time 4 seconds and amplitude 10 cm. [October, 1994]
(Ans: (i) after 0.5 second at the earliest, (ii) at 7.07 cm on either side of the mean position)

12) A particle of 10 gm executes S. H. M. of periodic time 6 seconds. One second after it has passed through its mean position, its velocity is found to be 6 cm /s. Calculate its mechanical energy. [March, 1994]
(Ans: 720 erg)

13) In a S. H. M., the value of maximum velocity is 10 m /s and the value of maximum acceleration is 20π m /s^2. Find the periodic time. [March, 1993]
(Ans: 1 second)

14) The acceleration of a particle executing S. H. M. is $-20\pi^2$ cm / s^2. Its time period is 0.4 second and its amplitude is 20 mm. Calculate the velocity and displacement.
(Ans: 28.8 cm /s, 0.8 cm) [October, 1992]

15) A simple harmonic oscillator starts motion from the mean position. Its periodic time is 8 sec. Find the time at the end of which its kinetic energy will become equal to half of its mechanical energy. [March, 1992]
(Ans: earliest after 1 second from the start of motion)

16) A glass tube of U-shape is partly filled with mercury of density d. Height of mercury column in each limb is L. If free surface of mercury in one of the limbs is made to oscillate by giving displacement y, prove that the oscillations are simple harmonic.
[October, 1990]

17) The acceleration of a particle is 150 m/s^2 when its displacement in a S. H. M. is 15 mm. Calculate its periodic time and frequency. [October, 1989]
(Ans: 0.063 s, 16 Hz)

18) A particle executing a S. H. M. has a maximum displacement of 4 cm and its acceleration at a distance of 1 cm from its mean position is 3 cm /s^2. What will be its velocity when it is at a distance of 2 cm from its mean position ? [March, 1989]
(Ans: 6 cm /s)

19) The equation of velocity of a simple harmonic oscillator is $v = 6\pi \cos(\pi t + \pi/6)$.
Find the equations for its displacement and acceleration. [March, 1988]
[Ans: $y = 6 \sin(\pi t + \pi/6)$, $a = -6\pi^2 \sin(\pi t + \pi/6)$]

20) The acceleration of the particle executing S. H. M. is $\pi^2/200$ m /s^2, when its displacement is 0.02 m. Determine its period. [October, 1986]
(Ans: 4 seconds)

21) For a simple harmonic oscillator, the value of amplitude, mass and angular frequency are respectively 10 cm, 400 gm and 6 Hz. Find the force constant of the spring.
(Ans: 568 N / m) (Note: It should be frequency and not angular frequency.) [May, 1986]

22) When displacements of an oscillator performing S. H. M. are y_1 and y_2, its velocities are v_1 and v_2 respectively. Prove that its amplitude and periodic time are given by

$$A = \left[\frac{v_1^2 y_2^2 - v_2^2 y_1^2}{v_1^2 - v_2^2}\right]^{\frac{1}{2}} \quad \text{and} \quad T = 2\pi\left[\frac{y_2^2 - y_1^2}{v_1^2 - v_2^2}\right]^{\frac{1}{2}}$$

23) The displacement in cm of an oscillator performing S. H. M. at instant 't' is given by $y = 5 \sin (10 \pi t + \pi / 6)$. Find (i) amplitude, (ii) periodic time, (iii) initial phase, (iv) displacement after 0.2 second, (v) velocity at 0.2 second and (vi) acceleration after 0.2 second.
(Ans: (i) 5 cm, (ii) 0.2 s, (iii) $\pi / 6$, (iv) 2.5 cm, (v) 136 cm / s, (vi) 24.65 m / s^2)

24) The maximum velocity of an oscillator performing S. H. M. is $20\sqrt{3}$ cm / s . What will be its velocity when it is midway between mid-position and positive end.
(Ans: 30 cm / s)

25) Find initial phase and amplitude for simple harmonic motion represented by an equation $y = 3 \cos \omega t + 4 \sin \omega t$.
(Ans: $36^\circ 52'$, 5 units)

26) A wooden rod of mass M and cross section A floats vertically in the liquid of density d. Its centre of mass is inside the liquid surface. If it is slightly pressed down and released, prove that it performs S. H. M. Find its periodic time.
[Ans: $T = 2 \pi (M / Adg)^{1/2}$]

27) A S. H. O. oscillates with periodic time of 6 seconds. Find the time elapsed when its phase changes from $\pi / 6$ to $5 \pi / 6$.
(Ans: 2 seconds)

28) If a block of mass 100 gm is suspended at the end of an elastic spring then its length increases by 9.8 cm. From this equilibrium condition, it is displaced upwards by 10 cm at time t = 0 and released. Obtain the displacement equation of its S. H. M.
[Ans: $y = 0.1 \sin (10t + \pi /2)$]

29) A block of mass 1 Kg is suspended to a spring having spring constant 100 N /m. It performs damped oscillations in a medium having damping coefficient 12 Kg /s. Find its angular frequency of oscillations.
(Ans: 8.0 rad /s)

30) One S. H. O. with mass 0.5 kg performs damped S. H. M. in a medium having damping coefficient 75 g /s. Find out the time taken for the amplitude to reduce by 25 % of initial amplitude. Also find the time at the end of which the mechanical energy becomes one fourth its initial value.
(Ans: 3.84 s, 9.24 s)

31) For a damped S. H. O., m = 2 kg, k = 10 N / m. If its amplitude becomes 3 /4 th of its initial value at the end of 4 oscillations, find the damping coefficient for this S. H. O. Consider $b / 2m << k / m$.
(Ans: 0.102 kg / s)

32) Derive differential equation of damped oscillations from its solution.