7 - ROTATIONAL MOTION

1) The minute's hand of a watch is 2 cm long. Find (a) the angular velocity, (b) linear velocity and (c) the radial acceleration. [April, 2002]
[Ans: (a) $\pi / 1800$ rad /s, (b) $\pi / 900$ cm /s, (c) $(\pi^2 / 162) \times 10^{-4}$ cm /s^2]
(Note: Linear velocity and radial acceleration are for a particle on the tip of the minute's hand of the watch.)

2) The length of a simple pendulum is one metre and mass of its bob is 10 gram. If the angular displacement is 36 °, find the angular acceleration. [October, 1998]
(Ans: 5.76 rad /s^2)

3) A force $\vec{F} = 2\hat{i} + 2\hat{j} + \hat{k}$ is acting at a point $\vec{r} = 2\hat{i} + 2\hat{j} + \hat{k}$ away from the axis of rotation. Find the torque acting on it. [March, 1998]
(Ans: zero)

4) The position vector of a particle is $\vec{r} = 2\hat{i} + 2\hat{j} + \hat{k}$ and the force acting on is $\vec{F} = 5\hat{i} + 6\hat{j} + 7\hat{k}$. Calculate the torque acting on the particle. [October, 1997]
(Ans: $8\hat{i} - 9\hat{j} + 2\hat{k}$)

5) What should be the maximum speed of a vehicle when it passes on the inclined curved path ? The radius of the curved path is 200 metre and slope $\theta = 11° 32'$. (The path is frictionless). [March, 1997]
(Ans: 20 m / s)

(Note: The problem is incorrectly framed. The path cannot be frictionless. The car cannot run on a frictionless path. At a very low speed, the car will have a tendency to slide inwards on the inclined path resulting in outward radial frictional force on its tyres. As the speed increases, this outward radial frictional force reduces and becomes zero at a definite speed. The answer given is this definite speed at which there is no radial frictional force on the tyres of the car and which the author intends to ask. If the speed of the car increases further, car will have a tendency to skid outwards and inward radial frictional force acts on its tyres which increases with speed of the car. At a certain maximum speed of the car, this frictional force reaches a maximum value. If the speed of the car exceeds this maximum value, it will skid outwards, assuming that the outward toppling speed is higher than this speed which is true for a vehicle like car having less height compared its width.)

6) A body is performing rotational motion. The motion is described by the equation $\theta = a t + b t^2$. Calculate the values of a and b in terms of initial angular speed (ω_0) and constant angular acceleration (α) derivating the above equation with respect to time. If a = 2 rad /s and b = 5 rad /s^2, compute ω_0 and α. [October, 1996]
(Ans: $a = \omega_0$, $b = \alpha / 2$, $\omega_0 = 2$ rad /s , $\alpha = 10$ rad /s^2)

7) A sphere, in space, revolves by 1 revolution in 10 seconds with respect to axis passing through its centre. Suddenly the diameter of the sphere becomes four times during the motion. How many revolutions will it perform in next 10 seconds ? [March, 1996]
(Ans: 1 / 16 revolution)

8) A circular disc of mass m and radius r has its moment of inertia $m r^2 / 2$. The disc is set rolling on a table. If ω is the angular velocity, show that its total kinetic energy is given by $(3 / 4) m r^2 \omega^2$. [March, 1995]

9) A rigid body is rotating with uniform angular acceleration. Its angular velocity and angular displacement after 2 seconds are 84 rad /s and 134 rad respectively. Find its angular displacement after 8 seconds. [October, 1994]
(Ans: 944 rad)

10) The angular velocity of a flywheel increases from 1200 rev /min in 10 seconds. Obtain its angular acceleration and the number of revolutions made during this time. [Oct., 1993]
(Ans: 3 rev /s^2, 350 revolutions)

11) A circular ring of radius 10 cm and mass 1 kg is rotating with angular velocity of 10 rev /s about the axis at right angles to its plane and passing through its centre. Find the work that must be done by a torque to increase the rate to 20 rev /s . [March, 1994]
(Ans: 59.2 J)

12) A flywheel takes 4 seconds to rotate through an angle of 240 rad. If its angular velocity at the end of this time becomes 80 rad /s, calculate its constant angular acceleration.
(Ans: 10 rad /s^2) [March, 1993]

13) Linear speed of a vehicle is 36 km /hr when angular acceleration of its wheel is 5 rad /s^2. Calculate its linear speed at the end of 5 s. The diameter of its wheel is 40 cm.
(Ans: 15 m /s) [October, 1992]

14) Find the angular velocity and acceleration of second, minute and hour hands of a clock.
(Ans: $\pi / 30$ rad /s, $\pi / 1800$ rad /s, $\pi / 21600$ rad /s ; $\alpha = 0$ for all) [March, 1992]

15) If the radius of the earth suddenly changes to X times the present value, find the period of revolution in terms of X. Also find the ratio of final angular energy to the initial angular kinetic energy in terms of X. Moment of inertia of earth = $(2 / 5) MR^2$.
(Ans: 24 X^2 hours, 1 / X^2) [October, 1991]

16) A solid cylinder of 5 kg and diameter 80 cm rolls down without slipping from the top of the frictionless inclined plane of height 4.9 m. Find its moment of inertia and velocity at the bottom of the slope. G = 9.8 m / s^2. [March, 1991]
(Ans: 0.40 Kg-m^2 about its axis of rolling, 8 m / s. Note: The statement 'frictionless plane' is incorrect as the cylinder cannot roll without slipping if the plane is frictionless.)

17) A ring of 5 metre diameter and 50000 gm mass is rotating about an axis which passes through its centre and perpendicular to its plane. The angular velocity of this ring is found to be increased from 5 rad /s to 25 rad /s in 5 seconds. Calculate the torque acting on this ring in M. K. S. [March, 1990]
(Ans: 1250 Nm)

18) A flywheel starts its motion with angular velocity of 2 rev /s and in 4 seconds it attains the angular velocity of 4 rev /s. Find its constant angular acceleration and angular displacement during this time. [October, 1989]
(Ans: 0.5 rev /s^2, 12 revolutions)

19) Moment of inertia of a flywheel about an axis passing through its centre and perpendicular to its plane is 62.5 kg-m^2. If its frequency is increased by 18 Hz in 5 seconds, calculate the torque acting on the flywheel. [March, 1989]
(Ans: 1413 Nm)

20) A ring of radius 40 cm and mass 60 kg rotates about an axis passing through its centre and perpendicular to its plane. The angular velocity of the ring increases from 5 rad /s to 25 rad /s in 5 sec. Calculate the work done by the torque in 5 sec.
[October, 1988, similar problem was asked in Oct., 1987 and May, 1986]
(Ans: 2880 J)

21) A ring of mass 1 kg and diameter 1 m slips from the top of a frictionless surface of the inclined plane of height 2 m. Find its moment of inertia and velocity at the bottom of the slope. [March, 1988]
(Ans: 0.25 kg-m^2 about an axis passing through its centre and perpendicular to its plane, 6.3 m /s)

22) Find the angular and linear velocity of the earth due to its revolution around the sun. Radius of earth's orbit = 1.5×10^8 km.
(Ans: 1.99×10^{-7} rad /s, 2.98×10^4 m /s)

23) A turntable rotates about a fixed vertical axis making one revolution in 10 seconds. The moment of inertia of the turntable about this axis is 1200 kg-m^2. A man of mass 80 kg initially standing at the centre of the turntable runs out along a radius. What is the angular velocity of the turntable when the man is 2 m from the centre.
(Ans: 0.5 rad /s)

24) A small block rests on a turntable at 0.65 m from the centre. It is rotated in such a way that the block undergoes a constant tangential acceleration, a_t = 2 m /s^2. Determine (a) how long will it take for the block to start slipping on the turntable, (b) the speed of the block at that instant. μ_s = 0.60.
[Ans: (a) 0.95 s, (b) 1.90 m /s]

25) An automobile track is so designed that when a car travels at 100 km /hour, the force between the car and the track acts normal to the surface of the track. Find the angle of banking the track assuming it to be a circle of radius 250 m.
(Ans: 17.46 °)

26) A car starts skidding when traveling at the speed of 60 km /hr on a horizontal road taking a curve of 50 m. At what angle should the road be banked so that the car can travel with the same speed of 60 km /hr without friction between the tyre and the road. What will be the maximum speed of the car on the banked road without skidding ?
(Ans: 29.55 °, 103 km /hr)

27) Length of a simple pendulum is 80 cm. Its maximum angular displacement in a vertical plane is 2 °. Mass of the bob is 0.1 kg. Determine the angular frequency, maximum velocity, maximum acceleration and maximum restoring force of the bob.
(Ans: 3.5 rad /s, 0.01 m /s, 0.34 m /s^2, 0.034 N)