15 - TRIGONOMETRY

(Answers at the end of all questions)

(1) In triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $a x^2 + b x + c = 0$, $a \neq 0$, then

(a) $a = b + c$ (b) $c = a + b$ (c) $b = c$ (d) $b = a + c$ [AIEEE 2005]

(2) In triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2 (r + R)$ equals

(a) $b + c$ (b) $a + b$ (c) $a + b + c$ (d) $c + a$ [AIEEE 2005]

(3) If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy \cos\alpha + y^2$ is equal to

(a) $2 \sin 2\alpha$ (b) 4 (c) $4 \sin^2 \alpha$ (d) $- 4 \sin^2 \alpha$ [AIEEE 2005]

(4) If in triangle ABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then sin A, sin B, sin C are in

(a) G.P. (b) A.P. (c) Arithmetic-Geometric Progression (d) H.P. [AIEEE 2005]

(5) Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$, then the value of $\cos\frac{\alpha - \beta}{2}$ is

(a) $-\frac{3}{\sqrt{130}}$ (b) $\frac{3}{\sqrt{130}}$ (c) $\frac{6}{65}$ (d) $-\frac{6}{65}$ [AIEEE 2004]

(6) If $u = \sin\sqrt{a^2 \cos^2\theta + b^2 \sin^2\theta} + \sqrt{a^2 \sin^2\theta + b^2 \sin^2\theta}$, then difference between the maximum and minimum values of u^2 is given by

(a) $2 (a^2 + b^2)$ (b) $2\sqrt{a^2 + b^2}$ (c) $(a + b)^2$ (d) $(a - b)^2$ [AIEEE 2004]

(7) The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1 + \sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is

(a) 60° (b) 90° (c) 120° (d) 150° [AIEEE 2004]

(8) A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of a river is 60° and when he retires 40 m away from the tree, the angle of elevation becomes 30°. The breadth of the river is

(a) 20 m (b) 30 m (c) 40 m (d) 60 m [AIEEE 2004]

(9) If in a triangle $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c are

(a) in A. P. (b) in G. P. (c) in H. P. (d) satisfy $a + b = c$ [AIEEE 2003]

(10) The sum of the radii of inscribed and circumscribed circles, for an n sided regular polygon of side a, is

(a) $a\cot\left(\frac{\pi}{2n}\right)$ (b) $b\cot\left(\frac{\pi}{n}\right)$ (c) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (d) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

[AIEEE 2003]

(11) The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. The height of the vertical pole is

(a) 20 m (b) 40 m (c) 60 m (d) 80 m [AIEEE 2003]

(12) The value of $\cos^2\alpha + \cos^2(\alpha + 120°) + \cos^2(\alpha - 120°)$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) 0 [AIEEE 2003]

(13) The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$ has a solution for

(a) $|a| < \frac{1}{\sqrt{2}}$ (b) $|a| \geq \frac{1}{\sqrt{2}}$ (c) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ (d) all real values of a

[AIEEE 2003]

(14) If $\sin\theta + \sin\phi = a$ and $\cos\theta + \cos\phi = b$, then the value of $\tan\left(\frac{\theta - \phi}{2}\right)$ is

(a) $\sqrt{\frac{a^2 + b^2}{4 - a^2 - b^2}}$ (b) $\sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$

(c) $\sqrt{\frac{a^2 + b^2}{4 + a^2 + b^2}}$ (d) $\sqrt{\frac{4 + a^2 + b^2}{a^2 + b^2}}$ [AIEEE 2002]

(15) If $\tan^{-1}(x) + 2\cot^{-1}(x) = \frac{2\pi}{3}$, then the value of x is

(a) $\sqrt{2}$ (b) 3 (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ [AIEEE 2002]

(16) The value of $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \ldots + \tan^{-1}\left(\frac{1}{n^2+n+1}\right)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) 0 [AIEEE 2002]

(17) The angles of elevation of the top of a tower (A) from the top (B) and bottom (D) at a building of height a are 30° and 45° respectively. If the tower and the building stand at the same level, then the height of the tower is

(a) $a\sqrt{3}$ (b) $\frac{a\sqrt{3}}{\sqrt{3}-1}$ (c) $\frac{a(3+\sqrt{3})}{2}$ (d) $a(\sqrt{3}-1)$ [AIEEE 2002]

(18) If $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$, $-\pi \le \alpha, \beta \le \pi$, then the number of ordered pairs (α, β) =

(a) 0 (b) 1 (c) 2 (d) 4 [IIT 2005]

(19) Which of the following is correct for triangle ABC having sides a, b, c opposite to the angles A, B, C respectively

(a) $a\sin\left(\frac{B-C}{2}\right) = (b-c)\cos\frac{A}{2}$ (b) $a\sin\left(\frac{B+C}{2}\right) = (b+c)\cos\frac{A}{2}$

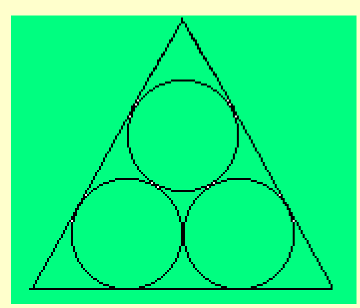
(c) $(b+c)\sin\left(\frac{B+C}{2}\right) = a\cos\frac{A}{2}$ (d) $\sin\left(\frac{B-C}{2}\right) = a\cos\frac{A}{2}$ [IIT 2005]

(20) Three circles of unit radii are inscribed in an equilateral triangle touching the sides of the triangle as shown in the figure. Then, the area of the triangle is

(a) $6 + 4\sqrt{3}$ (b) $12 + 8\sqrt{3}$

(c) $7 + 4\sqrt{3}$ (d) $4 + \frac{7}{2}\sqrt{3}$

[IIT 2005]



(Answers at the end of all questions)

(21) If θ and ϕ are acute angles such that $\sin\theta = \frac{1}{2}$ and $\cos\theta = \frac{1}{3}$, then θ and ϕ lies in

(a) $\left]\frac{\pi}{3}, \frac{\pi}{2}\right]$ (b) $\left]\frac{\pi}{2}, \frac{2\pi}{3}\right[$ (c) $\left]\frac{2\pi}{3}, \frac{5\pi}{3}\right[$ (d) $\left]\frac{5\pi}{6}, \pi\right[$ [IIT 2004]

(22) For which value of x, $\sin[\cot^{-1}(x + 1)] = \cos(\tan^{-1} x)$?

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) $-\frac{1}{2}$ [IIT 2004]

(23) If a, b, c are the sides of a triangle such that $a : b : c = 1 : \sqrt{3} : 2$, then $A : B : C$ is

(a) 3 : 2 : 1 (b) 3 : 1 : 2 (c) 1 : 3 : 2 (d) 1 : 2 : 3 [IIT 2004]

(24) Value of $\sqrt{x^2 + x} + \frac{\tan^2\alpha}{\sqrt{x^2 + x}}$, $x > 0$, $\alpha \in \left(0, \frac{\pi}{2}\right)$ is always greater than or equal to

(a) 2 (b) $\frac{5}{2}$ (c) $2\tan\alpha$ (d) $\sec\alpha$ [IIT 2003]

(25) If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the largest side to the perimeter is equal to

(a) $1 : 1 + \sqrt{3}$ (b) 2 : 3 (c) $\sqrt{3} : 2 + \sqrt{3}$ (d) $1 : 2 + \sqrt{3}$ [IIT 2003]

(26) The natural domain of $\sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for all $x \in R$, is

(a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $\left[-\frac{1}{2}, \frac{1}{4}\right]$ [IIT 2003]

(27) The length of a longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π [IIT 2002]

(28) Which of the following pieces of data does NOT uniquely determine an acute-angled triangle ABC (R being the radius of the circumcircle) ?

(a) a sin A, sin B (b) a, b, c (c) a, sin B, R (d) a, sin A, R [IIT 2002]

(29) The number of integral values of k for which the equation $7 \cos x + 5 \sin x = 2k + 1$ has a solution is

(a) 4 (b) 8 (c) 10 (d) 12 [IIT 2002]

(30) Let $0 < \alpha < \frac{\pi}{2}$ be a fixed angle. If $P = (\cos \theta, \sin \theta)$ and $Q = [\cos (\alpha - \theta), \sin (\alpha - \theta)]$, then Q is obtained from P by

(a) clockwise rotation around origin through an angle α
(b) anticlockwise rotation around origin through an angle α
(c) reflection in the line through origin with slope $\tan \alpha$
(d) reflection in the line through origin with slope $\tan \frac{\alpha}{2}$ [IIT 2002]

(31) Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals

(a) $\sqrt{PQ \cdot RS}$ (b) $\frac{PQ + RS}{2}$ (c) $\frac{2 PQ \cdot RS}{PQ + RS}$ (d) $\sqrt{\frac{PQ^2 + RS^2}{2}}$ [IIT 2001]

(32) A man from the top of a 100 metres high tower sees a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance in (metres) traveled by the car during this time is

(a) $100 \sqrt{3}$ (b) $\frac{200 \sqrt{3}}{3}$ (c) $\frac{100 \sqrt{3}}{3}$ (d) $200 \sqrt{3}$ [IIT 2001]

(33) If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then $\tan \alpha$ equals

(a) $2 (\tan \beta + \tan \gamma)$ (b) $\tan \beta + \tan \gamma$
(c) $\tan \beta + 2 \tan \gamma$ (d) $2 \tan \beta + \tan \gamma$ [IIT 2001]

(34) If $\sin^{-1} \left(x - \frac{x^2}{2} + \frac{x^3}{4} - \dots \right) + \cos^{-1} \left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \dots \right) = \frac{\pi}{2}$ for $0 < | x | < \sqrt{2}$, then x equals

(a) $\frac{1}{2}$ (b) 1 (c) $- \frac{1}{2}$ (d) - 1 [IIT 2001]

(35) The maximum value of $(\cos \alpha_1) \cdot (\cos \alpha_2)$ ..... $(\cos \alpha_n)$, under the restrictions $0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \frac{\pi}{2}$ and $(\cos \alpha_1) \cdot (\cos \alpha_2)$ ..... $(\cos \alpha_n) = 1$ is

(a) $\frac{1}{2^{n/2}}$ (b) $\frac{1}{2^n}$ (c) $\frac{1}{2n}$ (d) 1 [IIT 2001]

(36) The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

(a) 0 (b) 2 (c) 1 (d) 3 [IIT 2001]

(37) If $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$, then $f(\theta)$

(a) ≥ 0 only when $\theta \ge 0$ (b) ≤ 0 for all real θ
(c) ≥ 0 for all real θ (d) ≤ 0 only when $\theta \le 0$ [IIT 2000]

(38) In a triangle ABC, $2ac \sin \frac{1}{2}(A - B + C) =$

(a) $a^2 + b^2 - c^2$ (b) $c^2 + a^2 - b^2$ (c) $b^2 - c^2 - a^2$ (d) $c^2 - a^2 - b^2$ [IIT 2000]

(39) In a triangle ABC, if $\angle C = \frac{\pi}{2}$, r = inradius and R = circum-radius, then $2(r + R) =$

(a) a + b (b) b + c (c) c + a (d) a + b + c [IIT 2000]

(40) A pole stands vertically inside a triangular park ΔABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in ΔABC, the foot of the pole is at the

(a) centroid (b) circumcentre (c) incentre (d) orthocentre [IIT 2000]

(41) In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$), then

(a) $a + b = c$ (b) $b + c = a$ (c) $c + a = b$ (d) $b = c$ [IIT 1999]

(42) The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$ is

(a) zero (b) one (c) two (d) infinite [IIT 1999]

(43) The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2x})$ attains its maximum is

(a) 0 (b) 1 (c) 2 (d) infinite [IIT 1998]

(44) If, for a positive integer n,

$$f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1 + \sec\theta)(1 + \sec 2\theta) \ldots (1 + \sec 2^n\theta), \text{ then}$$

(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$

(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$ [IIT 1999]

(45) If in a triangle PQR, sin P, sin Q, sin R are in A. P., then

(a) the altitudes are in A. P. (b) the altitudes are in H. P.
(c) the medians are in G. P. (d) the medians are in A. P. [IIT 1998]

(46) The number of values of x in the interval [0, 5π] satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is

(a) 0 (b) 5 (c) 6 (d) 10 [IIT 1998]

(47) Which of the following number(s) is / are rational ?

(a) sin 15° (b) cos 15° (c) sin 15° cos 15° (d) sin 15° cos 75° [IIT 1998]

(48) Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then b_0 and b_1 respectively are

(a) 1, 3 (b) 0, n (c) - 1, n (d) 0, n^2 - 3n + 3 [IIT 1998]

(49) The parameter, on which the value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$$ does not depend upon is

(a) a (b) p (c) d (d) x [IIT 1997]

(50) The graph of the function $\cos x \cos(x+2) - \cos^2(x+1)$ is

(a) a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to the X-axis

(b) a straight line passing through (0, $-\sin^2 1$) with slope 2

(c) a straight line passing through (0, 0)

(d) a parabola with vertex (1, $-\sin^2 1$) [IIT 1997]

(51) If $A_0 A_1 A_2 A_3 A_4 A_5$ be a regular hexagon inscribed in a circle of unit radius, then the product of the lengths of the line segments $A_0 A_1$, $A_0 A_2$ and $A_0 A_4$ is

(a) $\frac{3}{4}$ (b) $3\sqrt{3}$ (c) 3 (d) $\frac{3\sqrt{3}}{2}$ [IIT 1998]

(52) $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if

(a) $x + y \neq 0$ (b) $x = y, x \neq 0$ (c) $x = y$ (d) $x \neq 0, y \neq 0$ [IIT 1996]

(53) The minimum value of the expression $\sin\alpha + \sin\beta + \sin\gamma$, where α, β, γ are the real numbers satisfying $\alpha + \beta + \gamma = \pi$ is

(a) positive (b) zero (c) negative (D) - 3 [IIT 1995]

(54) In a triangle ABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$. If D divides $\overline{BC}$ internally in the ratio 1 : 3, then $\frac{\sin \angle BAD}{\sin \angle CAD}$ equals

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{\frac{2}{3}}$ [IIT 1995]

(55) Number of solutions of the equation $\tan x + \sec x = 2 \cos x$, lying in the interval $[0, 2\pi]$, is

(a) 0 (b) 1 (c) 2 (d) 3 [IIT 1993]

(56) If $x = \sum_{n=0}^{\infty} \cos^{2n} \phi$, $y = \sum_{n=0}^{\infty} \sin^{2n} \phi$, $z = \sum_{n=0}^{\infty} \cos^{2n} \phi \sin^{2n} \phi$, for $0 < \phi < \frac{\pi}{2}$, then

(a) $xyz = xz + y$ (b) $xyz = xy + z$
(c) $xyz = x + y + z$ (d) $xyz = yz + x$ [IIT 1993]

(57) If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where $[x]$ stands for the greatest integer function, then

(a) $f\left(\frac{\pi}{2}\right) = -1$ (b) $f(\pi) = 1$ (c) $f(-\pi) = 0$ (d) $f\left(\frac{\pi}{4}\right) = 2$ [IIT 1991]

(58) The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ in the variable x has real roots. Then p can take any value in the interval

(a) $(0, 2\pi)$ (b) $(-\pi, 0)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$ [IIT 1990]

(59) In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $3 \sin x - 4 \sin^3 x - k = 0$, $0 < k < 1$, then the measure of angle C is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$ [IIT 1990]

(60) The number of real solutions of the equation $\sin(e^x) = 5^x + 5^{-x}$ is

(a) 0 (b) 1 (c) 2 (d) infinitely many [IIT 1990]

(61) The general solution of $\sin x - 3\sin 2x + \sin 3x = \cos x - \cos 2x + \cos 3x$ is

(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$

(c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (d) $2n\pi + \cos^{-1}\frac{3}{2}$ [IIT 1989]

(62) The value of the expression $\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ$ is equal to

(a) 2 (b) 4 (c) $\frac{2\sin 20^\circ}{\sin 40^\circ}$ (d) $\frac{4\sin 20^\circ}{\sin 40^\circ}$ [IIT 1988]

(63) The values of θ lying between $\theta = 0$ and $\theta = \frac{\pi}{2}$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ are}$$

(a) $\frac{7\pi}{24}$ (b) $\frac{5\pi}{24}$ (c) $\frac{11\pi}{24}$ (d) $\frac{\pi}{24}$ [IIT 1988]

(64) In a triangle, the lengths of the two larger sides are 10 and 9 respectively. If the angles are in A. P., then the lengths of the third side can be

(a) $5 - \sqrt{6}$ (b) $3\sqrt{3}$ (c) 5 (d) $5 + \sqrt{6}$ [IIT 1987]

(65) The smallest positive root of the equation $\tan x = x$ lies in

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \pi\right)$ (c) $\left(\pi, \frac{3\pi}{2}\right)$ (d) $\left(\pi, \frac{3\pi}{2}\right)$ [IIT 1987]

(66) The number of all triplets (a_1, a_2, a_3) such that $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for all x is

(a) 0 (b) 1 (c) 3 (d) infinite (e) none of these [IIT 1987]

(67) The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

(a) $-\frac{2\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{3}$ (e) none of these [IIT 1986]

(68) The expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$ is equal to

(a) 0 (b) 1 (c) 3 (d) $\sin 4\alpha + \cos 4\alpha$ (e) none of these [IIT 1986]

(69) There exists a triangle ABC satisfying the conditions

(a) $b\sin A = a$, $A < \frac{\pi}{2}$ (b) $b\sin A > a$, $A > \frac{\pi}{2}$

(c) $b\sin A > a$, $A < \frac{\pi}{2}$ (d) $b\sin A < a$, $A < \frac{\pi}{2}$, $b > a$

(e) $b\sin A < a$, $A > \frac{\pi}{2}$, $b = a$ [IIT 1986]

(70) $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is equal to

(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$ (c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$ [IIT 1984]

(71) From the top of a light-house 60 m high with its base at the sea-level, the angle of depression of a boat is 15°. The distance of the boat from the foot of the lighthouse is

(a) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)60$ metres (b) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$ metres

(c) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)60$ metres (d) None of these [IIT 1983]

(72) The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$ is

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$ (c) $\frac{16}{7}$ (d) None of these [IIT 1983]

(73) If $f(x) = \cos(\ln x)$, then $f(x) f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy) \right]$ has the value

(a) -1 (b) $\frac{1}{2}$ (c) -2 (d) none of these [IIT 1983]

(74) The general solution of the trigonometric equation $\sin x + \cos x = 1$ is given by

(a) $x = 2n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ (b) $x = 2n\pi + \frac{\pi}{2}$, $n = 0, \pm 1, \pm 2, \ldots$

(c) $x + n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$, $n = 0, \pm 1, \pm 2, \ldots$ (d) none of these [IIT 1981]

(75) If $A = \sin^2\theta + \cos^4\theta$, then for all real values of θ

(a) $1 \leq A \leq 2$ (b) $\frac{3}{4} \leq A \leq 1$

(c) $\frac{13}{16} \leq A \leq 1$ (d) $\frac{3}{4} \leq A \leq \frac{13}{16}$ [IIT 1980]

(76) The equation $2\cos^2\left(\frac{1}{2}x\right)\sin^2 x = x^2 + x^{-2}$, $0 < x \leq \frac{\pi}{2}$ has

(a) no real solution (b) one real solution
(c) more than one real solution [IIT 1980]

(77) If $\tan\theta = \frac{-4}{3}$, then $\sin\theta$ is

(a) $\frac{-4}{5}$ but not $\frac{4}{5}$ (b) $\frac{-4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $\frac{-4}{5}$ (d) none of these [IIT 1979]

(78) If $\alpha + \beta + \gamma = 2\pi$, then

(a) $\tan\frac{\gamma}{2} + \tan\frac{\beta}{2} + \tan\frac{\alpha}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$

(c) $\tan\frac{\gamma}{2} + \tan\frac{\beta}{2} + \tan\frac{\alpha}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(d) none of these [IIT 1979]

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	b	c	b	a	d	c	a	a	c	b	a	a	b	c	b	c	d	a	a

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	d	d	c	c	a	a	d	b	d	a	b	c	b	a	c	c	b	a	b

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
a	c	a	a,b,c,d	d	c	c	b	b	a	c	b	c	a	d	b	a,c	b	c	0

61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
b	b	a,c	a,c	a	d	e	b	a,d	c	c	d	d	c	b	a	b	a		