12 - THREE DIMENSIONAL GEOMETRY

(Answers at the end of all questions)

(1) If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}\,x + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$, then the value of λ is

(a) $\frac{5}{3}$ (b) $-\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $-\frac{4}{3}$ [AIEEE 2005]

(2) If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then a equals

(a) -1 (b) 1 (c) -2 (d) 2 [AIEEE 2005]

(3) The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is

(a) $\frac{10}{9}$ (b) $\frac{10}{3\sqrt{3}}$ (c) $\frac{3}{10}$ (d) $\frac{10}{3}$ [AIEEE 2005]

(4) The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) $0°$ (b) $90°$ (c) $45°$ (d) $30°$ [AIEEE 2005]

(5) The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius

(a) 3 (b) 1 (c) 2 (d) $\sqrt{2}$ [AIEEE 2005]

(6) A line makes the same angle θ with each of the X- and Z- axis. If the angle β, which it makes with the y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

(a) $\frac{2}{3}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$ [AIEEE 2004]

(7) Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{9}{2}$ [AIEEE 2004]

(8) A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The coordinates of each of the points of intersection are given by

(a) (3a, 3a, 3a), (a, a, a) (b) (3a, 2a, 3a), (a, a, a)
(c) (3a, 2a, 3a), (a, a, 2a) (d) (2a, 3a, 3a), (2a, a, a) [AIEEE 2004]

(9) If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively, are co-planar, then λ equals

(a) -2 (b) -1 (c) $-\frac{1}{2}$ (d) 0 [AIEEE 2004]

(10) The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the spheres and the plane

(a) $x - y - z = 1$ (b) $x - 2y - z = 1$
(c) $x - y - 2z = 1$ (d) $2x - y - z = 1$ [AIEEE 2004]

(11) The lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ will be perpendicular if and only if

(a) $aa' + cc' + 1 = 0$ (b) $aa' + cc' = 0$
(c) $aa' + bb' = 0$ and (d) $aa' + bb' + cc' = 0$ [AIEEE 2003]

(12) The lines $\frac{x - 2}{1} = \frac{y - 3}{1} = \frac{z - 4}{-k}$ and $\frac{x - 1}{k} = \frac{y - 4}{2} = \frac{z - 5}{1}$ are coplanar, if

(a) $k = 0$ or -1 (b) $k = 1$ or -1
(c) $k = 0$ or -3 (d) $k = 3$ or -3 [AIEEE 2003]

(13) Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a', b' c' from the origin, then

(a) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = 0$ (b) $\frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

(c) $\frac{1}{a^2} - \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$ (d) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

[AIEEE 2003]

(14) The direction cosines of the normal to the plane $x + 2y - 3z + 4 = 0$ are

(a) $-\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (b) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

(c) $-\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (d) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}$ [AIEEE 2003]

(15) The radius of a circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z = 19$ is cut by the plane $x + 2y + 2z + 7 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4 [AIEEE 2003]

(16) The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

(a) 13 (b) 26 (c) 39 (d) 11 [AIEEE 2003]

(17) The distance of a point (1, - 2, 3) from the plane $x - y + z = 5$ and parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

(a) 1 (b) 7 (c) 3 (d) 13 [AIEEE 2002]

(18) The co-ordinates of the point in which the line joining the points (3, 5, - 7) and (- 2, 1, 8) and intersected by the YZ-plane are

(a) $\left(0, \frac{13}{5}, 2 \right)$ (b) $\left(0, -\frac{13}{5}, -2 \right)$

(c) $\left(0, -\frac{13}{5}, \frac{2}{5} \right)$ (d) $\left(0, \frac{13}{5}, \frac{2}{5} \right)$ [AIEEE 2002]

(19) The angle between the planes $2x - y + 3z = 6$ and $x + y + 2z = 7$ is

(a) $0°$ (b) $30°$ (c) $45°$ (d) $60°$ [AIEEE 2002]

(20) If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are at right angles, then the value of k is

(a) $-\frac{10}{7}$ (b) $-\frac{7}{10}$ (c) -10 (d) -7 [AIEEE 2002]

(21) A unit vector perpendicular to the plane of $\vec{a} = 2\vec{i} - 6\vec{j} - 3\vec{k}$ and $\vec{b} = 4\vec{i} + 3\vec{j} - \vec{k}$ is

(a) $\frac{4\vec{i} + 3\vec{j} - \vec{k}}{\sqrt{26}}$ (b) $\frac{2\vec{i} - 6\vec{j} - 3\vec{k}}{7}$

(c) $\frac{3\vec{i} - 2\vec{j} + 6\vec{k}}{7}$ (d) $\frac{2\vec{i} - 3\vec{j} - 6\vec{k}}{7}$ [AIEEE 2002]

(22) A unit vector normal to the plane through the points $\vec{i}$, $2\vec{j}$ and $3\vec{k}$ is

(a) $6\vec{i} + 3\vec{j} + 2\vec{k}$ (b) $\vec{i} + 2\vec{j} + 3\vec{k}$

(c) $\frac{6\vec{i} + 3\vec{j} + 2\vec{k}}{7}$ (d) $\left| \frac{6\vec{i} + 3\vec{j} + 2\vec{k}}{7} \right|$ [AIEEE 2002]

(23) A plane at a unit distance from the origin intersects the coordinate axes at P, Q and R. If the locus of the centroid of Δ PQR satisfies the equation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, then the value of k is

(a) 1 (b) 3 (c) 6 (d) 9 [IIT 2005]

(24) Two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect at a point, then k is

(a) $\frac{3}{2}$ (b) $\frac{9}{2}$ (c) $\frac{2}{9}$ (d) 2 [IIT 2004]

(25) If the line $\frac{x - 1}{1} = \frac{y - 2}{1} = \frac{z - k}{2}$ lies exactly on the plane $2x - 4y + z = 7$, then the value of k is

(a) 7 (b) - 7 (c) 1 (d) no real value [IIT 2003]

(26) There are infinite planes passing through the points (3, 6, 7) touching the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z = 11$. If the plane passing through the circle of contact cuts intercepts a, b, c on the co-ordinate axes, then a + b + c =

(a) 12 (b) 23 (c) 67 (d) 47

(27) The mid-points of the chords cut off by the lines through the point (3, 6, 7) intersecting the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z = 11$ lie on a sphere whose radius =

(a) 3 (b) 4 (c) 5 (d) 6

(28) The ratio of magnitudes of total surface area to volume of a right circular cone with vertex at origin, having semi-vertical angle equal to $30°$ and the circular base on the plane $x + y + z = 6$ is

(a) 1 (b) 2 (c) 3 (d) 4

(29) The direction of normal to the plane passing through origin and the line of intersection of the planes $x + 2y + 3z = 4$ and $4x + 3y + 2z = 1$ is

(a) (1, 2, 3) (b) (3, 2, 1) (c) (2, 3, 1) (d) (3, 1, 2)

(30) The volume of the double cone having vertices at the centres of the spheres $x^2 + y^2 + z^2 = 25$ and $x^2 + y^2 + z^2 - 4x - 8y - 8z + 11 = 0$ and the common circle of the spheres as the circular base of the double cone is

(a) 24π (b) 32π (c) 28π (d) 36π

(31) A line through the point P (0, 6, 8) intersects the sphere $x^2 + y^2 + z^2 = 36$ in points A and B. $PA \times PB$ =

(a) 36 (b) 24 (c) 100 (d) 64

(32) A sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0$ is inscribed in a cone with vertex at (6, 6, 6). The semi-vertical angle of the cone is

(a) 15° (b) 30° (c) 45° (d) 60°

(33) The point which is farthest on the sphere $x^2 + y^2 + z^2 = 144$ from the point (2, 4, 4) is

(a) (3, 6, 6) (b) (- 3, - 6, - 6) (c) (4, 8, 8) (d) (- 4, - 8, - 8)

(34) The equation of the plane containing the line $x + y - z = 0 = 2x - y + 4$ and passing through the point (1, 1, 1) is

(a) $3x + 4y - 5z = 2$ (b) $4x + 5y - 6z = 3$
(c) $x + y + z = 3$ (d) $3x + 6y - 5z = 4$

(35) A plane passes through the points of intersection of the spheres $x^2 + y^2 + z^2 = 36$ and $x^2 + y^2 + z^2 - 4x - 4y - 8z - 12 = 0$. A line joining the centres of the spheres intersects this plane at

(a) (1, 1, 1) (b) (1, 1, 2) (c) (1, 2, 1) (d) (2, 1, 1)

(36) The area of the circle formed by the intersection of the spheres $x^2 + y^2 + z^2 = 36$ and $x^2 + y^2 + z^2 - 4x - 4y - 8z - 12 = 0$ is

(a) 9π (b) 18π (c) 27π (d) 36π

(37) A line joining the points (1, 1, 1) and (2, 2, 2) intersects the plane $x + y + z = 9$ at the point

(a) (3, 4, 2) (b) (2, 3, 4) (c) (3, 2, 4) (d) (3, 3, 3)

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	c	b	b	b	c	c	b	a	d	a	c	d	d	c	a	a	a	d	a

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	c	d	b	a	d	a	c	b	b	d	c	d	d	b	c	d			