9 - INTEGRAL CALCULUS

(Answers at the end of all questions)

(1) If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$, $I_4 = \int_1^2 2^{x^3} dx$, then

(a) $I_2 > I_1$ (b) $I_1 > I_2$ (c) $I_3 = I_4$ (d) $I_3 > I_4$ [AIEEE 2005]

(2) The area enclosed between the curve $y = \log_e (x + e)$ and the coordinate axes is

(a) 1 (b) 2 (c) 3 (d) 4 [AIEEE 2005]

(3) The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the area of these parts numbered from top to bottom, then $S_1 : S_2 : S_3$ is

(a) 1 : 2 : 1 (b) 1 : 3 : 1 (c) 2 : 1 : 2 (d) 1 : 1 : 1 [AIEEE 2005]

(4) $\int \left\{ \dfrac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$ is equal to

(a) $\dfrac{\log x}{(\log x)^2 + 1} + c$ (b) $\dfrac{x}{x^2 + 1} + c$

(c) $\dfrac{xe^x}{1 + x^2} + c$ (d) $\dfrac{x}{(\log x)^2 + 1} + c$ [AIEEE 2005]

(5) Let $f (x)$ be a non-negative continuous function such that the area bounded by the curve $y = f (x)$, X-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $(\beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta)$. Then $f (\frac{\pi}{2})$ is

(a) $\frac{\pi}{4} + \sqrt{2} - 1$ (b) $\frac{\pi}{4} - \sqrt{2} + 1$

(c) $1 - \frac{\pi}{4} - \sqrt{2}$ (d) $1 - \frac{\pi}{4} + \sqrt{2}$ [AIEEE 2005]

(6) The value of $\int_{-\pi}^{\pi} \dfrac{\cos^2 x}{1 + a^x} dx$, $a > 0$ is

(a) $a\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{a}$ (d) 2π [AIEEE 2005]

(7) $\lim_{n \to \infty} \sum_{r=1}^{n} e^{\frac{r}{n}}$ is

(a) e (b) e - 1 (c) 1 - e (d) e + 1 [AIEEE 2004]

(8) If $\int \frac{\sin x}{\sin(x - \alpha)} dx = Ax + B \log \sin(x - \alpha) + C$, then the value of (A, B) is

(a) $(\sin\alpha, \cos\alpha)$ (b) $(\cos\alpha, \sin\alpha)$
(c) $(-\sin\alpha, \cos\alpha)$ (d) $(-\cos\alpha, \sin\alpha)$ [AIEEE 2004]

(9) $\int \frac{dx}{\cos x - \sin x}$ is equal to

(a) $\frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} - \frac{\pi}{8}\right) \right| + C$ (b) $\frac{1}{\sqrt{2}} \log \left| \cot\left(\frac{x}{2}\right) \right| + C$
(c) $\frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} - \frac{3\pi}{8}\right) \right| + C$ (d) $\frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} + \frac{3\pi}{8}\right) \right| + C$ [AIEEE 2004]

(10) The value of $\int_{-2}^{3} \left| 1 - x^2 \right| dx$ is

(a) $\frac{28}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{1}{3}$ [AIEEE 2004]

(11) The value of $I = \int_{0}^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is

(a) 0 (b) 1 (c) 2 (d) 3 [AIEEE 2004]

(12) If $\int_{0}^{\pi} x\, f(\sin x)\, dx = A \int_{0}^{\frac{\pi}{2}} f(\sin x)\, dx$, then A is equal to

(a) 0 (b) π (c) $\frac{\pi}{4}$ (d) 2π [AIEEE 2004]

(13) If $f(x) = \dfrac{e^x}{1 + e^x}$, $I_1 = \displaystyle\int_{f(-a)}^{f(a)} x\,g\{x(1-x)\}\,dx$ and $I_2 = \displaystyle\int_{f(-a)}^{f(a)} g\{x(1-x)\}\,dx$, then the value of $\dfrac{I_2}{I_1}$ is

(a) 2 (b) - 3 (c) - 1 (d) 1 [AIEEE 2004]

(14) The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and X-axis is

(a) 1 (b) 2 (c) 3 (d) 4 [AIEEE 2004]

(15) The value of $\displaystyle\lim_{x \to 0} \frac{\int_0^{x^2} \sec^2 t \, dt}{x \sin x}$ is

(a) 3 (b) 2 (c) 1 (d) 0 [AIEEE 2003]

(16) The value of the integral $I = \displaystyle\int_0^1 x(1-x)^n \, dx$ is

(a) $\dfrac{1}{n+1}$ (b) $\dfrac{1}{n+2}$ (c) $\dfrac{1}{n+1} - \dfrac{1}{n+2}$ (d) $\dfrac{1}{n+1} + \dfrac{1}{n+2}$ [AIEEE 2003]

(17) If $f(y) = e^y$, $g(y) = y$, $y > 0$ and $F(t) = \displaystyle\int_0^t f(t-y)\,g(y)\,dy$, then

(a) $F(t) = t e^t$ (b) $F(t) = t e^{-t}$
(c) $F(t) = e^t - (1 + t)$ (d) $F(t) = 1 - e^{-t}(1 + t)$ [AIEEE 2003]

(18) If $f(a + b - x) = f(x)$, then the value of $\displaystyle\int_a^b x f(x)\,dx$ is

(a) $\dfrac{b-a}{2}\displaystyle\int_a^b f(x)\,dx$ (b) $\dfrac{a+b}{2}\displaystyle\int_a^b f(x)\,dx$

(c) $\dfrac{a+b}{2}\displaystyle\int_a^b f(b-x)\,dx$ (d) $\displaystyle\int_a^b f(a+b-x)\,dx$ [AIEEE 2003]

(19) Let $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(K) - F(1)$, then one of the possible values of K is

(a) 15 (b) 16 (c) 63 (d) 64 [AIEEE 2003]

(20) Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. The value of the integral $\int_0^1 f(x)\,g(x)\,dx$ is

(a) $e - \frac{e^2}{2} - \frac{5}{2}$ (b) $e + \frac{e^2}{2} - \frac{3}{2}$

(c) $e - \frac{e^2}{2} - \frac{3}{2}$ (d) $e + \frac{e^2}{2} + \frac{5}{2}$ [AIEEE 2003]

(21) If $\int x \sin x \, dx = -x \cos x + \alpha$, then the value of α is

(a) $\sin x + c$ (b) $\cos x + c$

(c) $x \cos x + c$ (d) $\cos x - \sin x + c$ [AIEEE 2002]

(22) The value of $\int \frac{1 - \cos 2x}{\cos 2x + 1} dx$ is

(a) $\tan x - x + c$ (b) $x + \tan x + c$

(c) $x - \tan x + c$ (d) $-x - \cot x + c$ [AIEEE 2002]

(23) The value of $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ is

(a) πab (b) $\pi^2 ab$ (c) π / ab (d) $\pi / 2ab$ [AIEEE 2002]

(24) The value of $\int e^{3 \log x} (x^4 + 1)^{-1} dx$ is

(a) $\log(x^4 + 1) + c$ (b) $\frac{1}{4} \log(x^4 + 1) + c$

(c) $3 \log(x^4 + 1) + c$ (d) $- \log(x^4 + 1) + c$ [AIEEE 2002]

(25) The value of $\int \frac{\log x}{x^2} dx$ is

(a) $\log(x + 1) + c$ (b) $-\frac{1}{x}\log(x + 1) + c$

(c) $\log(x - 1) + c$ (d) $\frac{1}{2}\log(x + 1) + c$ [AIEEE 2002]

(26) The value of $\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t})\, dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t})\, dt$ is

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{\pi}{4}$ (d) π [AIEEE 2002]

(27) If the area bounded by the X-axis, the curve $y = f(x)$ and the lines $x = 1$, $x = b$ is equal to $\sqrt{b^2 + 1} - \sqrt{2}$ for all $b > 1$, then $f(x)$ is

(a) $\sqrt{x - 1}$ (b) $\sqrt{x + 1}$ (c) $\sqrt{x^2 + 1}$ (d) $\frac{x}{\sqrt{1 + x^2}}$ [AIEEE 2002]

(28) $\int_{-2}^{0} \left[x^3 + 3x^2 + 3x + 3 + (x + 1)\cos(x + 1)\right] dx =$

(a) 4 (b) 0 (c) - 1 (d) 1 [IIT 2005]

(29) Find the area between the curves $y = (x - 1)^2$, $y = (x + 1)^2$ and $y = \frac{1}{4}$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{1}{6}$ [IIT 2005]

(30) If $\int_{\sin x}^{1} t^2 f(t)\, dt = 1 - \sin x$, $x \in [0, \pi/2]$, then $f(1/\sqrt{3})$ is

(a) 3 (b) 1 / 3 (c) 1 (d) $\sqrt{3}$ [IIT 2005]

(31) If $\int_0^{t^2} x f(x)\, dx = \frac{2}{5}t^5$ for $t > 0$, then $f\left(\frac{4}{25}\right)$ is

(a) $-\frac{2}{5}$ (b) 0 (c) $\frac{2}{5}$ (d) 1 [IIT 2004]

(32) $\int_0^1 \sqrt{\frac{1 - x}{1 + x}}\, dx$ is equal to

(a) $\frac{\pi}{2} + 1$ (b) $\frac{\pi}{2} - 1$ (c) 1 (d) π [IIT 2004]

(33) If the area bounded by the curves $x = ay^2$ and $y = ax^2$ is 1, then a is equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 3 [IIT 2004]

(34) If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2}\, dt$, then the interval in which $f(x)$ is increasing is

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $[-2, 2]$ (d) nowhere [IIT 2003]

(35) If $I(m, n) = \int_0^1 t^m (1 + t)^n\, dt$, $m, n \in R$, then $I(m, n)$ is

(a) $\frac{n}{1 + m} I[(m + 1), (n - 1)]$ (b) $\frac{2^n}{1+m} - \frac{m}{1 + n} I[(m + 1), (n - 1)]$

(c) $\frac{2^n}{1+m} - \frac{m}{1 + m} I[(m + 1), (n - 1)]$ (d) $\frac{m}{n+1} I[(m + 1), (n - 1)]$ [IIT 2003]

(36) Area bounded by the curves $y = \sqrt{x}$, $x = 2y + 3$ in the first quadrant and X-axis is

(a) $2\sqrt{3}$ (b) 18 (c) 9 (d) $\frac{34}{3}$ [IIT 2003]

(37) The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) 4 [IIT 2002]

(38) If $f(x) = \int_1^x \sqrt{2 - t^2}\, dt$, then the real roots of the equation $x^2 - f'(x) = 0$ are

(a) ± 1 (b) $\pm \frac{1}{\sqrt{2}}$ (c) $\pm \frac{1}{2}$ (d) 0 and 1 [IIT 2002]

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(Answers at the end of all questions)

(39) Let $T > 0$ be a fixed real number. Suppose f is a continuous function such that for all $x \in R$, $f(x + T) = f(x)$. If $I = \int_0^T f(x)\,dx$, then the value of $\int_3^{3+3T} f(2x)\,dx$ is

(a) $\frac{3}{2}I$ (b) I (c) $3I$ (d) $6I$ [IIT 2002]

(40) The integral equals $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([\,x\,] + \ln\left(\frac{1+x}{1-x}\right)\right) dx$ equals

(a) $-\frac{1}{2}$ (b) 0 (c) 1 (d) $2\ln\frac{1}{2}$ [IIT 2002]

(41) If $f : (0, \infty) \rightarrow R$, $F(x) = \int_0^x f(t)\,dt$ and $F(x^2) = x^2(1 + x)$, then $f(4)$ equals

(a) $\frac{5}{4}$ (b) 7 (c) 4 (d) 2 [IIT 2001]

(42) The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x}\,dx$, $a > 0$, is

(a) π (b) $a\pi$ (c) $\frac{\pi}{2}$ (d) 2π [IIT 2001]

(43) If $f(x) = \begin{cases} e^{\cos x}\sin x & \text{for } |x| \leq 2, \\ 2 & \text{otherwise,} \end{cases}$ then $\int_{-2}^{3} f(x)\,dx =$

(a) 0 (b) 1 (c) 2 (d) 3 [IIT 2000]

(44) The value of the integral $\int_{e^{-1}}^{e^2} \left| \frac{\log_e x}{x} \right| dx$ is

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5 [IIT 2000]

(Answers at the end of all questions)

(45) If $f(x) = \int e^{x}(x-1)(x-2)\,dx$, then f decreases in the interval

(a) $(-\infty, -2)$ (b) $(-2, -1)$ (c) $(1, 2)$ (d) $(2, +\infty)$ [IIT 2000]

(46) Let $g(x) = \int_{0}^{x} f(t)\,dt$, where f is such that $\frac{1}{2} \le f(t) \le 1$ for $t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for $t \in [1, 2]$. Then $g(2)$ satisfies the inequality

(a) $-\frac{3}{2} \le g(2) \le \frac{1}{2}$ (b) $0 \le g(2) \le 2$

(c) $\frac{3}{2} \le g(2) \le \frac{5}{2}$ (d) $2 < g(2) < 4$ [IIT 2000]

(47) If for a real number y, [y] is the greatest integer less than or equal to y, then the value of the integral $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2\sin x]\,dx$ is

(a) $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$ [IIT 1999]

(48) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x} =$

(a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ [IIT 1999]

(49) For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$?

(a) -4 (b) -2 (c) 2 (d) 4 [IIT 1999]

(50) If $f(x) = x - [x]$, for every real number x, where [x] is the integral part of x, then $\int_{-1}^{1} f(x)\,dx$ is

(a) 1 (b) 2 (c) 0 (d) $\frac{1}{2}$ [IIT 1998]

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(Answers at the end of all questions)

(51) If $g(x) = \int_0^x \cos^4 t \, dt$, then $g(x + \pi)$ equals

(a) $g(x) + g(\pi)$ (b) $g(x) - g(\pi)$ (c) $g(x)\,g(\pi)$ (d) $\dfrac{g(x)}{g(\pi)}$ [IIT 1997]

(52) Let f be a positive function. If $I_1 = \int_{1-k}^{k} x\, f[x(1-x)]\,dx$ and $I_2 = \int_{1-k}^{k} f[x(1-x)]\,dx$, where $2k - 1 > 0$, then $\dfrac{I_1}{I_2}$ is

(a) 2 (b) k (c) $\dfrac{1}{2}$ (d) 1 [IIT 1997]

(53) The slope of the tangent to a curve $y = f(x)$ at $[x, f(x)]$ is $2x + 1$. If the curve passes through the point (1, 2), then the area of the region bounded by the curve, the X-axis and the line $x = 1$ is

(a) $\dfrac{5}{6}$ (b) $\dfrac{6}{5}$ (c) $\dfrac{1}{6}$ (d) 6 [IIT 1995]

(54) The value of $\int_{\pi}^{2\pi} [2\sin x]\,dx$ where [.] represents the greatest integer function, is

(a) $-\dfrac{5\pi}{3}$ (b) $-\pi$ (c) $\dfrac{5\pi}{3}$ (d) -2π [IIT 1995]

(55) The value of $\int_0^{\frac{\pi}{2}} \dfrac{dx}{1 + \tan^3 x}$ is

(a) 0 (b) 1 (c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{4}$ [IIT 1993]

(56) If $f : R \rightarrow R$ be a differentiable function and $f(1) = 4$, then the value of $\lim\limits_{x \to 1} \int_4^{f(x)} \dfrac{2t}{x - 1}\,dt$ is

(a) $8f'(1)$ (b) $4f'(1)$ (c) $2f'(1)$ (d) $f'(1)$ [IIT 1990]

(57) If $f : R \to R$ and $g : R \to R$ are continuous functions, then the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) + g(-x)]\,dx$ is

(a) π (b) 1 (c) - 1 (d) 0 [IIT 1990]

(58) For any integer n, the integral $\int_0^{\pi} e^{\cos^2 x} \cos^3 (2n + 1) x \, dx$ has the value

(a) π (b) 1 (c) 0 (d) none of these [IIT 1985]

(59) The value of the integral $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) none of these [IIT 1983]

(60) If the area bounded by the curves $y = f(x)$, the X-axis and the ordinates $x = 1$ and $x = b$ is $(b - 1)\sin(3b + 4)$, then $f(x)$ is

(a) $(x - 1)\cos(3x + 4)$ (b) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$
(c) $\sin(3x + 4)$ (d) none of these [IIT 1982]

(61) The value of the definite integral $\int_0^1 (1 + e^{-x^2})\,dx$ is

(a) - 1 (b) 2 (c) $1 + e^{-1}$ (d) none of these [IIT 1981]

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(Answers at the end of all questions)

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	a	d	d	d	b	b	b	a,d	a	2	b	a	a	c	c	c	b	d	b
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	c	d	b	b	c	d	a	a	a	c	b	a	b	c	b	b	a	c	a
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c	c	c	b	c	b	c	a	b,d	a	a	c	a	a	d	a	d	c	a	b
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
d																			