

9 - INTEGRAL CALCULUS
(Answers at the end of all questions)

(1) If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$, $I_4 = \int_1^2 2^{x^3} dx$, then

- (a) $I_2 > I_1$ (b) $I_1 > I_2$ (c) $I_3 = I_4$ (d) $I_3 > I_4$ [AIEEE 2005]

(2) The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is

- (a) 1 (b) 2 (c) 3 (d) 4 [AIEEE 2005]

(3) The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the area of these parts numbered from top to bottom, then $S_1 : S_2 : S_3$ is

- (a) 1 : 2 : 1 (b) 1 : 3 : 1 (c) 2 : 1 : 2 (d) 1 : 1 : 1 [AIEEE 2005]

(4) $\int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$ is equal to

- (a) $\frac{\log x}{(\log x)^2 + 1} + c$ (b) $\frac{x}{x^2 + 1} + c$
(c) $\frac{xe^x}{1 + x^2} + c$ (d) $\frac{x}{(\log x)^2 + 1} + c$ [AIEEE 2005]

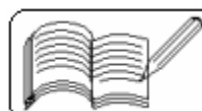
(5) Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, X-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is

$(\beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta)$. Then $f(\frac{\pi}{2})$ is

- (a) $\frac{\pi}{4} + \sqrt{2} - 1$ (b) $\frac{\pi}{4} - \sqrt{2} + 1$
(c) $1 - \frac{\pi}{4} - \sqrt{2}$ (d) $1 - \frac{\pi}{4} + \sqrt{2}$ [AIEEE 2005]

(6) The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx$, $a > 0$ is

- (a) $a\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{a}$ (d) 2π [AIEEE 2005]



(7) $\lim_{n \rightarrow \infty} \sum_{r=1}^n e^{\frac{r}{n}}$ is

- (a) e (b) $e - 1$ (c) $1 - e$ (d) $e + 1$

[AIEEE 2004]

(8) If $\int \frac{\sin x}{\sin(x - \alpha)} dx = Ax + B \log \sin(x - \alpha) + C$, then the value of (A, B) is

- (a) $(\sin \alpha, \cos \alpha)$ (b) $(\cos \alpha, \sin \alpha)$
(c) $(-\sin \alpha, \cos \alpha)$ (d) $(-\cos \alpha, \sin \alpha)$

[AIEEE 2004]

(9) $\int \frac{dx}{\cos x - \sin x}$ is equal to

- (a) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{\pi}{8} \right) \right| + C$ (b) $\frac{1}{\sqrt{2}} \log \left| \cot \left(\frac{x}{2} \right) \right| + C$
(c) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{3\pi}{8} \right) \right| + C$ (d) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + C$

[AIEEE 2004]

(10) The value of $\int_{-2}^3 |1 - x^2| dx$ is

- (a) $\frac{28}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{1}{3}$

[AIEEE 2004]

(11) The value of $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is

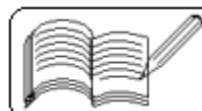
- (a) 0 (b) 1 (c) 2 (d) 3

[AIEEE 2004]

(12) If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is equal to

- (a) 0 (b) π (c) $\frac{\pi}{4}$ (d) 2π

[AIEEE 2004]



9 - INTEGRAL CALCULUS
(Answers at the end of all questions)

(13) If $f(x) = \frac{e^x}{1 + e^x}$, $I_1 = \int_{f(-a)}^{f(a)} x g\{x(1-x)\} dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\} dx$,

then the value of $\frac{I_2}{I_1}$ is

- (a) 2 (b) -3 (c) -1 (d) 1

[AIEEE 2004]

(14) The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and X-axis is

- (a) 1 (b) 2 (c) 3 (d) 4

[AIEEE 2004]

(15) The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$ is

- (a) 3 (b) 2 (c) 1 (d) 0

[AIEEE 2003]

(16) The value of the integral $I = \int_0^1 x(1-x)^n dx$ is

- (a) $\frac{1}{n+1}$ (b) $\frac{1}{n+2}$ (c) $\frac{1}{n+1} - \frac{1}{n+2}$ (d) $\frac{1}{n+1} + \frac{1}{n+2}$

[AIEEE 2003]

(17) If $f(y) = e^y$, $g(y) = y$, $y > 0$ and $F(t) = \int_0^t f(t-y)g(y)dy$, then

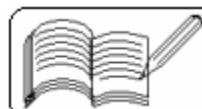
- (a) $F(t) = te^t$ (b) $F(t) = te^{-t}$
(c) $F(t) = e^t - (1+t)$ (d) $F(t) = 1 - e^{-t}(1+t)$

[AIEEE 2003]

(18) If $f(a+b-x) = f(x)$, then the value of $\int_a^b xf(x) dx$ is

- (a) $\frac{b-a}{2} \int_a^b f(x) dx$ (b) $\frac{a+b}{2} \int_a^b f(x) dx$
(c) $\frac{a+b}{2} \int_a^b f(b-x) dx$ (d) $\int_a^b f(a+b-x) dx$

[AIEEE 2003]



9 - INTEGRAL CALCULUS
(Answers at the end of all questions)

(19) Let $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(K) - F(1)$, then one of the possible values of K is

- (a) 15 (b) 16 (c) 63 (d) 64 [AIEEE 2003]

(20) Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. The value of the integral $\int_0^1 f(x)g(x) dx$ is

- (a) $e - \frac{e^2}{2} - \frac{5}{2}$ (b) $e + \frac{e^2}{2} - \frac{3}{2}$
(c) $e - \frac{e^2}{2} - \frac{3}{2}$ (d) $e + \frac{e^2}{2} + \frac{5}{2}$ [AIEEE 2003]

(21) If $\int x \sin x dx = -x \cos x + \alpha$, then the value of α is

- (a) $\sin x + c$ (b) $\cos x + c$
(c) $x \cos x + c$ (d) $\cos x - \sin x + c$ [AIEEE 2002]

(22) The value of $\int \frac{1 - \cos 2x}{\cos 2x + 1} dx$ is

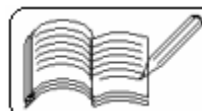
- (a) $\tan x - x + c$ (b) $x + \tan x + c$
(c) $x - \tan x + c$ (d) $-x - \cot x + c$ [AIEEE 2002]

(23) The value of $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ is

- (a) πab (b) $\pi^2 ab$ (c) π/ab (d) $\pi/2ab$ [AIEEE 2002]

(24) The value of $\int e^{3 \log x} (x^4 + 1)^{-1} dx$ is

- (a) $\log(x^4 + 1) + c$ (b) $\frac{1}{4} \log(x^4 + 1) + c$
(c) $3 \log(x^4 + 1) + c$ (d) $-\log(x^4 + 1) + c$ [AIEEE 2002]



(25) The value of $\int \frac{\log x}{x^2} dx$ is

- (a) $\log(x + 1) + c$ (b) $-\frac{1}{x} \log(x + 1) + c$
(c) $\log(x - 1) + c$ (d) $\frac{1}{2} \log(x + 1) + c$

[AIEEE 2002]

(26) The value of $\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t}) dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t}) dt$ is

- (a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{\pi}{4}$ (d) π

[AIEEE 2002]

(27) If the area bounded by the X-axis, the curve $y = f(x)$ and the lines $x = 1, x = b$ is equal to $\sqrt{b^2 + 1} - \sqrt{2}$ for all $b > 1$, then $f(x)$ is

- (a) $\sqrt{x - 1}$ (b) $\sqrt{x + 1}$ (c) $\sqrt{x^2 + 1}$ (d) $\frac{x}{\sqrt{1 + x^2}}$

[AIEEE 2002]

(28) $\int_{-2}^0 [x^3 + 3x^2 + 3x + 3 + (x + 1)\cos(x + 1)] dx =$

- (a) 4 (b) 0 (c) -1 (d) 1

[IIT 2005]

(29) Find the area between the curves $y = (x - 1)^2, y = (x + 1)^2$ and $y = \frac{1}{4}$

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{1}{6}$

[IIT 2005]

(30) If $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x, x \in [0, \pi/2]$, then $f(1/\sqrt{3})$ is

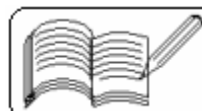
- (a) 3 (b) 1/3 (c) 1 (d) $\sqrt{3}$

[IIT 2005]

(31) If $\int_0^{t^2} x f(x) dx = \frac{2}{5} t^5$ for $t > 0$, then $f\left(\frac{4}{25}\right)$ is

- (a) $-\frac{2}{5}$ (b) 0 (c) $\frac{2}{5}$ (d) 1

[IIT 2004]



(32) $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ is equal to

- (a) $\frac{\pi}{2} + 1$ (b) $\frac{\pi}{2} - 1$ (c) 1 (d) π [IIT 2004]

(33) If the area bounded by the curves $x = ay^2$ and $y = ax^2$ is 1, then a is equal to

- (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 3 [IIT 2004]

(34) If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$, then the interval in which $f(x)$ is increasing is

- (a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $[-2, 2]$ (d) nowhere [IIT 2003]

(35) If $I(m, n) = \int_0^1 t^m (1+t)^n dt$, $m, n \in \mathbb{R}$, then $I(m, n)$ is

- (a) $\frac{n}{1+m} I[(m+1), (n-1)]$ (b) $\frac{2^n}{1+m} - \frac{m}{1+n} I[(m+1), (n-1)]$
(c) $\frac{2^n}{1+m} - \frac{m}{1+m} I[(m+1), (n-1)]$ (d) $\frac{m}{n+1} I[(m+1), (n-1)]$ [IIT 2003]

(36) Area bounded by the curves $y = \sqrt{x}$, $x = 2y + 3$ in the first quadrant and X-axis is

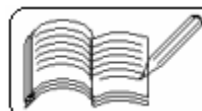
- (a) $2\sqrt{3}$ (b) 18 (c) 9 (d) $\frac{34}{3}$ [IIT 2003]

(37) The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

- (a) 1 (b) 2 (c) $2\sqrt{2}$ (d) 4 [IIT 2002]

(38) If $f(x) = \int_1^x \sqrt{2-t^2} dt$, then the real roots of the equation $x^2 - f'(x) = 0$ are

- (a) ± 1 (b) $\pm \frac{1}{\sqrt{2}}$ (c) $\pm \frac{1}{2}$ (d) 0 and 1 [IIT 2002]



(39) Let $T > 0$ be a fixed real number. Suppose f is a continuous function such that for all $x \in \mathbb{R}$, $f(x + T) = f(x)$. If $I = \int_0^T f(x) dx$, then the value of $\int_3^{3+3T} f(2x) dx$ is

- (a) $\frac{3}{2}I$ (b) I (c) $3I$ (d) $6I$ [IIT 2002]

(40) The integral equals $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x] + \ln\left(\frac{1+x}{1-x}\right) \right) dx$ equals

- (a) $-\frac{1}{2}$ (b) 0 (c) 1 (d) $2 \ln \frac{1}{2}$ [IIT 2002]

(41) If $f: (0, \infty) \rightarrow \mathbb{R}$, $F(x) = \int_0^x f(t) dt$ and $F(x^2) = x^2(1+x)$, then $f(4)$ equals

- (a) $\frac{5}{4}$ (b) 7 (c) 4 (d) 2 [IIT 2001]

(42) The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, $a > 0$, is

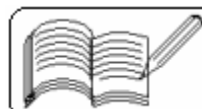
- (a) π (b) $a\pi$ (c) $\frac{\pi}{2}$ (d) 2π [IIT 2001]

(43) If $f(x) = \begin{cases} e^{\cos x} \sin x & \text{for } |x| \leq 2, \\ 2 & \text{otherwise,} \end{cases}$ then $\int_{-2}^3 f(x) dx =$

- (a) 0 (b) 1 (c) 2 (d) 3 [IIT 2000]

(44) The value of the integral $\int_{e^{-1}}^{e^2} \left| \frac{\log_e x}{x} \right| dx$ is

- (a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5 [IIT 2000]



(45) If $f(x) = \int e^x(x-1)(x-2)dx$, then f decreases in the interval

- (a) $(-\infty, -2)$ (b) $(-2, -1)$ (c) $(1, 2)$ (d) $(2, +\infty)$ [IIT 2000]

(46) Let $g(x) = \int_0^x f(t)dt$, where f is such that $\frac{1}{2} \leq f(t) \leq 1$ for $t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for $t \in [1, 2]$. Then $g(2)$ satisfies the inequality

- (a) $-\frac{3}{2} \leq g(2) \leq \frac{1}{2}$ (b) $0 \leq g(2) \leq 2$
(c) $\frac{3}{2} \leq g(2) \leq \frac{5}{2}$ (d) $2 < g(2) < 4$ [IIT 2000]

(47) If for a real number y , $[y]$ is the greatest integer less than or equal to y , then the value of the integral $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2 \sin x] dx$ is

- (a) $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$ [IIT 1999]

(48) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x} =$

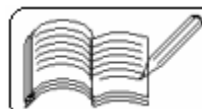
- (a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ [IIT 1999]

(49) For which of the following values of m , is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$?

- (a) -4 (b) -2 (c) 2 (d) 4 [IIT 1999]

(50) If $f(x) = x - [x]$, for every real number x , where $[x]$ is the integral part of x , then $\int_{-1}^1 f(x) dx$ is

- (a) 1 (b) 2 (c) 0 (d) $\frac{1}{2}$ [IIT 1998]



(51) If $g(x) = \int_0^x \cos^4 t \, dt$, then $g(x + \pi)$ equals

- (a) $g(x) + g(\pi)$ (b) $g(x) - g(\pi)$ (c) $g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$ [IIT 1997]

(52) Let f be a positive function. If $I_1 = \int_{1-k}^k x f[x(1-x)] dx$ and $I_2 = \int_{1-k}^k f[x(1-x)] dx$,

where $2k - 1 > 0$, then $\frac{I_1}{I_2}$ is

- (a) 2 (b) k (c) $\frac{1}{2}$ (d) 1 [IIT 1997]

(53) The slope of the tangent to a curve $y = f(x)$ at $[x, f(x)]$ is $2x + 1$. If the curve passes through the point $(1, 2)$, then the area of the region bounded by the curve, the X-axis and the line $x = 1$ is

- (a) $\frac{5}{6}$ (b) $\frac{6}{5}$ (c) $\frac{1}{6}$ (d) 6 [IIT 1995]

(54) The value of $\int_{\pi}^{2\pi} [2 \sin x] dx$ where $[.]$ represents the greatest integer function, is

- (a) $-\frac{5\pi}{3}$ (b) $-\pi$ (c) $\frac{5\pi}{3}$ (d) -2π [IIT 1995]

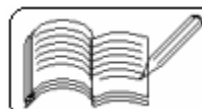
(55) The value of $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \tan^3 x}$ is

- (a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$ [IIT 1993]

(56) If $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function and $f(1) = 4$, then the value of

$\lim_{x \rightarrow 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is

- (a) $8f'(1)$ (b) $4f'(1)$ (c) $2f'(1)$ (d) $f'(1)$ [IIT 1990]



(57) If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions, then the value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) + g(-x)] dx \text{ is}$$

- (a) π (b) 1 (c) -1 (d) 0

[IIT 1990]

(58) For any integer n , the integral $\int_0^{\pi} e^{\cos^2 x} \cos^3 (2n + 1)x dx$ has the value

- (a) π (b) 1 (c) 0 (d) none of these

[IIT 1985]

(59) The value of the integral $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ is

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) none of these

[IIT 1983]

(60) If the area bounded by the curves $y = f(x)$, the X-axis and the ordinates $x = 1$ and $x = b$ is $(b - 1) \sin(3b + 4)$, then $f(x)$ is

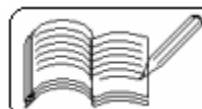
- (a) $(x - 1) \cos(3x + 4)$ (b) $\sin(3x + 4) + 3(x - 1) \cos(3x + 4)$
(c) $\sin(3x + 4)$ (d) none of these

[IIT 1982]

(61) The value of the definite integral $\int_0^1 (1 + e^{-x^2}) dx$ is

- (a) -1 (b) 2 (c) $1 + e^{-1}$ (d) none of these

[IIT 1981]



Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	a	d	d	d	b	b	b	a,d	a	2	b	a	a	c	c	c	b	d	b
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	c	d	b	b	c	d	a	a	a	c	b	a	b	c	b	b	a	c	a
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c	c	c	b	c	b	c	a	b,d	a	a	c	a	a	d	a	d	c	a	b
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
d																			

