

(1) $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right]$ is

- (a) $\frac{1}{2} \sec 1$ (b) $\frac{1}{2} \operatorname{cosec} 1$ (c) $\tan 1$ (d) $\frac{1}{2} \tan 1$ [AIEEE 2005]

(2) The normal to the curve $x = a(\cos \theta + \theta \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$ at any point 'θ' is such that

- (a) it passes through the origin
 (b) it makes angle $\frac{\pi}{2} + \theta$ with the X-axis
 (c) it passes through $(a\frac{\pi}{2}, -a)$
 (d) it is at a constant distance from the origin

[AIEEE 2005]

(3) A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched ?

<u>Interval</u>	<u>Function</u>	<u>Interval</u>	<u>Function</u>
(a) $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$	(b) $[2, \infty)$	$2x^3 - 3x^2 + 12x + 6$
(c) $(-\infty, \frac{1}{3})$	$3x^3 - 2x^2 + 1$	(d) $(-\infty, -4)$	$x^3 - 6x^2 + 6$

[AIEEE 2005]

(4) Let α and β be the distinct roots of the equation $ax^2 + bx + c = 0$. Then

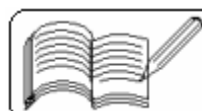
$\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to

- (a) $\frac{a^2}{2}(\alpha - \beta)^2$ (b) 0 (c) $-\frac{a^2}{2}(\alpha - \beta)^2$ (d) $\frac{1}{2}(\alpha - \beta)^2$

[AIEEE 2005]

(5) Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h \rightarrow 0} \frac{1}{h} f(1+h) = 5$, then $f'(1)$ equals

- (a) 3 (b) 4 (c) 5 (d) 6 [AIEEE 2005]



(6) Let f be differentiable for all x . If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then

- (a) $f(6) \geq 8$ (b) $f(6) < 8$ (c) $f(6) < 5$ (d) $f(6) = 5$ [AIEEE 2005]

(7) If f is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2$, $x, y \in \mathbb{R}$ and $f(0) = 0$, then $f(1)$ equals

- (a) -1 (b) 0 (c) 2 (d) 1 [AIEEE 2005]

(8) A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50 \text{ cm}^3/\text{min}$. When the thickness of ice is 5 cm, then the rate at which thickness of ice decreases in cm/min is

- (a) $\frac{1}{36\pi}$ (b) $\frac{1}{18\pi}$ (c) $\frac{1}{54\pi}$ (d) $\frac{5}{6\pi}$ [AIEEE 2005]

(9) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function having $f(2) = 6$, $f'(2) = \frac{1}{48}$. Then

$$\lim_{x \rightarrow 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt \text{ equals}$$

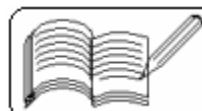
- (a) 24 (b) 36 (c) 12 (d) 18 [AIEEE 2005]

(10) If the equation $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$ has a positive root $x = \alpha$, then the equation $n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \dots + a_1 = 0$ has a positive root which is

- (a) greater than α (b) smaller than α
(c) greater than or equal to α (d) equal to α [AIEEE 2005]

(11) If $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b are

- (a) $a \in \mathbb{R}$, $b \in \mathbb{R}$ (b) $a = 1$, $b \in \mathbb{R}$
(c) $a \in \mathbb{R}$, $b = 2$ (d) $a = 1$, $b = 2$ [AIEEE 2004]



(12) Let $f(x) = \frac{1 - \tan x}{4x - \pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

- (a) 1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -1

[AIEEE 2004]

(13) If $x = e^y + e^{y+\dots+\infty}$, $x > 0$, then $\frac{dy}{dx}$ is

- (a) $\frac{x}{1+x}$ (b) $\frac{1}{x}$ (c) $\frac{1-x}{x}$ (d) $\frac{1+x}{x}$

[AIEEE 2004]

(14) A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

- (a) (2, 4) (b) (2, -4) (c) $\left(-\frac{9}{8}, \frac{9}{2}\right)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

[AIEEE 2004]

(15) A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is

- (a) $(x - 1)^2$ (b) $(x - 1)^3$ (c) $(x + 1)^3$ (d) $(x + 1)^2$

[AIEEE 2004]

(16) The normal to the curve $x = a(1 + \cos \theta)$, $y = a \sin \theta$ at ' θ ' always passes through the fixed point

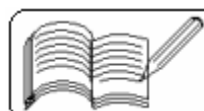
- (a) (a, 0) (b) (0, a) (c) (0, 0) (d) (a, a)

[AIEEE 2004]

(17) If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

- (a) (0, 1) (b) (1, 2) (c) (2, 3) (d) (1, 3)

[AIEEE 2004]



- (18) Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A. P., then $f'(a), f'(b)$ and $f'(c)$ are in
- (a) A. P. (b) G. P. (c) H. P. (d) A. G. P. [AIEEE 2003]

(19)
$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\left[1 - \tan\left(\frac{x}{2}\right)\right] [1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right)\right] [\pi - 2x]^3} =$$

(a) 0 (b) ∞ (c) $\frac{1}{32}$ (d) $\frac{1}{8}$ [AIEEE 2003]

(20) If $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, then the value of k is

(a) 0 (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$ [AIEEE 2003]

(21) If $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ is continuous at $x=0$, then the value of $f(0)$ is

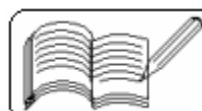
(a) ab (b) $a+b$ (c) $a-b$ (d) $\log a - \log b$ [AIEEE 2003]

(22) If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, then the value of $\frac{dy}{dx}$ is

(a) 0 (b) 1 (c) x (d) y [AIEEE 2003]

(23) The value of $\lim_{n \rightarrow \infty} \frac{1 + 2^4 + 3^4 + \dots + n^4}{n^5}$ is

(a) zero (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{30}$ [AIEEE 2003]



(24) If $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$ and $f(1) = 7$, then the value of $\sum_{r=1}^n f'(r)$ is

- (a) $\frac{7n}{2}$ (b) $7n(n + 1)$ (c) $\frac{7(n + 1)}{2}$ (d) $\frac{7n(n + 1)}{2}$ [AIEEE 2003]

(25) The real number x when added to its inverse gives the minimum value of the sum at x equal to

- (a) 2 (b) -2 (c) 1 (d) -1 [AIEEE 2003]

(26) If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

- (a) 3 (b) 1 (c) 2 (d) 4 [AIEEE 2003]

(27) If $f(x) = x^n$, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \frac{f'''(1)}{3!} + \dots + \frac{(-1)^n f^n(1)}{n!}$ is

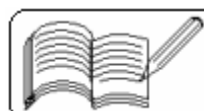
- (a) 2^n (b) 2^{n-1} (c) 1 (d) 0 [AIEEE 2003]

(28) If $x = t^2 + t + 1$ and $y = \sin\left(\frac{\pi}{2}t\right) + \cos\left(\frac{\pi}{2}t\right)$, then at $t = 1$, the value of $\frac{dy}{dx}$ is

- (a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{4}$ [AIEEE 2002]

(29) If $x = 3 \cos \theta - 2 \cos^3 \theta$ and $y = 3 \sin \theta - 2 \sin^3 \theta$, then the value of $\frac{dy}{dx}$ is

- (a) $\sin \theta$ (b) $\cos \theta$ (c) $\tan \theta$ (d) $\cot \theta$ [AIEEE 2002]



(30) Let $f(a) = g(a) = k$ and their n th derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n . Further if $\lim_{x \rightarrow a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$, then the value of k is

- (a) 4 (b) 2 (c) 1 (d) 0

[AIEEE 2002]

(31) The value of $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x) \sin 5x}{x^2 \sin 3x}$ is

- (a) $\frac{10}{3}$ (b) $\frac{3}{10}$ (c) $\frac{6}{5}$ (d) $\frac{5}{6}$

[AIEEE 2002]

(32) The value of $\lim_{\alpha \rightarrow \beta} \left[\frac{\sin^2 \alpha - \sin^2 \beta}{\alpha^2 - \beta^2} \right]$ is

- (a) 0 (b) 1 (c) $\frac{\sin \beta}{\beta}$ (d) $\frac{\sin 2\beta}{2\beta}$

[AIEEE 2002]

(33) The value of $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{x}$ is

- (a) 0 (b) 1 (c) $\sqrt{2}$ (d) does not exist

[AIEEE 2002]

(34) If $f(x) = 2x^3 - 3x^2 - 12x + 5$ on $[-2, 4]$, then relative maximum occurs at $x =$

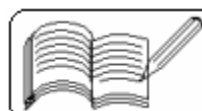
- (a) -2 (b) -1 (c) 2 (d) 4

[AIEEE 2002]

(35) If $f(x) = \begin{cases} x e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is

- (a) discontinuous everywhere
(b) continuous as well as differentiable for all x
(c) neither differentiable nor continuous at $x = 0$
(d) continuous at all x but not differentiable at $x = 0$

[AIEEE 2002]



(36) If y is a twice differentiable function and $x \cos y + y \cos x = \pi$, then $y''(0) =$

- (a) π (b) $-\pi$ (c) 0 (d) 1 [IIT 2005]

(37) $f(x) = ||x| - 1|$ is not differentiable at $x =$

- (a) 0, ± 1 (b) ± 1 (c) 0 (d) 1 [IIT 2005]

(38) If f is a differentiable function such that $f: \mathbb{R} \rightarrow \mathbb{R}$, $f\left(\frac{1}{n}\right) = 0 \quad \forall n \in \mathbb{I}, n \geq 1$, then

- (a) $f(x) = 0 \quad \forall x \in [0, 1]$ (b) $f(0) = 0$, but $f'(0)$ may or may not be 0
(c) $f(0) = 0 = f'(0)$ (d) $|f(x)| \leq 1 \quad \forall x \in [0, 1]$ [IIT 2005]

(39) f is a twice differentiable polynomial function of x such that $f(1) = 1$, $f(2) = 4$ and $f(3) = 9$, then

- (a) $f''(x) = 2, \quad \forall x \in \mathbb{R}$ (b) $f''(x) = f'(x) = 5, x \in [1, 3]$
(c) $f''(x) = 2$ for only $x \in [1, 3]$ (d) $f''(x) = 3, x \in (1, 3)$ [IIT 2005]

[Note: This question should have been better put as 'polynomial function of degree two rather than twice differentiable function'.]

(40) S is a set of polynomial of degree less than or equal to 2, $f(0) = 0$, $f(1) = 1$, $f'(x) > 0, \quad \forall x \in [0, 1]$, then set $S =$

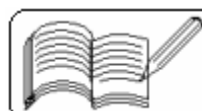
- (a) $ax + (1 - a)x^2, a \in \mathbb{R}$ (b) $ax + (1 - a)x^2, 0 < a < 2$
(c) $ax + (1 - a)x^2, 0 < a < \infty$ (d) ϕ [IIT 2005]

(41) Let y be a function of x , such that $\log(x + y) = 2xy$, then $y'(0)$ is

- (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$ [IIT 2004]

(42) Let $f(x) = x^\alpha \log x$ for $x > 0$ and $f(0) = 0$ follows Rolle's theorem for $x \in [0, 1]$, then α is

- (a) -2 (b) -1 (c) 0 (d) $\frac{1}{2}$ [IIT 2004]



(43) If $f(x)$ is strictly increasing and differentiable, then $\lim_{x \rightarrow 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is

- (a) 1 (b) -1 (c) 0 (d) 2

[IIT 2004]

(44) Let $f(x) = x^3 + bx^2 + cx + d$, $0 < b^2 < c$, then $f(x)$

- (a) is strictly increasing (b) has local maxima
(c) has local minima (d) is a bounded curve

[IIT 2004]

(45) If $f(x)$ is a differentiable function, $f'(1) = 1$, $f'(2) = 6$, where $f'(c)$ means the derivative of the function at $x = c$, then

$$\lim_{h \rightarrow 0} \frac{f(2 + 2h + h^2) - f(2)}{f(1 + h - h^2) - f(1)}$$

- (a) does not exist (b) -3 (c) 3 (d) $\frac{3}{2}$

[IIT 2003]

(46) If $\lim_{x \rightarrow 0} \frac{\sin nx [(a - n)nx - \tan x]}{x^2} = 0$, where n is a non-zero positive integer, then a is equal to

- (a) $\frac{n+1}{n}$ (b) n^2 (c) $\frac{1}{n}$ (d) $n + \frac{1}{n}$

[IIT 2003]

(47) Which function does not obey Mean Value Theorem in $[0, 1]$?

(a) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$ (b) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(c) $f(x) = x |x|$

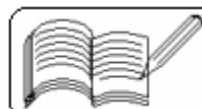
(d) $f(x) = |x|$

[IIT 2003]

(48) The domain of the derivative of the function $f(x) = \begin{cases} \tan^{-1} x, & \text{if } |x| \leq 1 \\ \frac{1}{2}(|x| - 1), & \text{if } |x| > 1 \end{cases}$ is

- (a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{-1\}$ (d) $\mathbb{R} - \{-1, 1\}$

[IIT 2002]



(49) The integer n for which $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is

- (a) 1 (b) 2 (c) 3 (d) 4 [IIT 2002]

(50) If $f: \mathbb{R} \rightarrow \mathbb{R}$ be such that $f(1) = 3$ and $f'(1) = 6$, then $\lim_{x \rightarrow 0} \left(\frac{f(1+x)}{f(1)} \right)^{\frac{1}{x}}$ equals

- (a) 1 (b) $e^{\frac{1}{2}}$ (c) e^2 (d) e^3 [IIT 2002]

(51) The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is/(are)

- (a) $\left(\pm \frac{4}{\sqrt{3}}, -2 \right)$ (b) $\left(\pm \sqrt{\frac{11}{3}}, 0 \right)$ (c) $(0, 0)$ (d) $\left(\pm \frac{4}{\sqrt{3}}, 2 \right)$ [IIT 2002]

(52) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \{x, x^3\}$. The set of all points where $f(x)$ is not differentiable is

- (a) $\{-1, 1\}$ (b) $\{-1, 0\}$ (c) $\{0, 1\}$ (d) $\{-1, 0, 1\}$ [IIT 2001]

(53) The left hand derivative of $f(x) = [x] \sin(\pi x)$ at $x = k$, where k is an integer, is

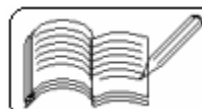
- (a) $(-1)^k (k-1)\pi$ (b) $(-1)^{k-1} (k-1)\pi$
(c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$ [IIT 2001]

(54) The left hand derivative of $f(x) = [x] \sin(\pi x)$ at $x = k$, where k is an integer, is

- (a) $(-1)^k (k-1)\pi$ (b) $(-1)^{k-1} (k-1)\pi$
(c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$ [IIT 2001]

(55) $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals

- (a) $-\pi$ (b) π (c) $\pi/2$ (d) 1 [IIT 2001]



(56) If $f(x) = x e^{x(1-x)}$, then $f(x)$ is

- (a) increasing on $\left[-\frac{1}{2}, 1\right]$ (b) decreasing on \mathbb{R}
(c) increasing on \mathbb{R} (d) decreasing on $\left[-\frac{1}{2}, 1\right]$ [IIT 2001]

(57) Which of the following functions is differentiable at $x = 0$?

- (a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$
(c) $\sin(|x|) + |x|$ (d) $\sin(|x|) - |x|$ [IIT 2001]

(58) If $x^2 + y^2 = 1$, then

- (a) $yy'' - 2(y')^2 + 1 = 0$ (b) $yy'' + (y')^2 + 1 = 0$
(c) $yy'' + (y')^2 - 1 = 0$ (d) $yy'' + 2(y')^2 + 1 = 0$ [IIT 2000]

(59) For $x \in \mathbb{R}$, $\lim_{x \rightarrow \infty} \left(\frac{x-3}{x+2}\right)^x =$

- (a) e (b) e^{-1} (c) e^{-5} (d) e^5 [IIT 2000]

(60) Consider the following statements in S and R :

S: Both $\sin x$ and $\cos x$ are decreasing functions in the interval $\left(\frac{\pi}{2}, \pi\right)$

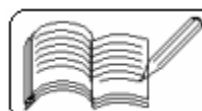
R: If a differentiable function decreases in an interval (a, b) , then its derivative also decreases in (a, b) .

Which of the following is true ?

- (a) Both S and R are wrong.
(b) Both S and R are correct, but R is not the correct explanation of S .
(c) S is correct and R is correct explanation of S .
(d) S is correct and R is wrong. [IIT 2000]

(61) If the normal to the curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive X -axis, then $f'(3) =$

- (a) -1 (b) $-3/4$ (c) $4/3$ (d) 1 [IIT 2000]



(62) If $f(x) = \begin{cases} |x| & \text{for } 0 < |x| \leq 2 \\ 1 & \text{for } x = 0 \end{cases}$, then at $x = 0$, f has

- (a) a local maximum (b) no local maximum
(c) a local minimum (d) no extremum

[IIT 2000]

(63) For all $x \in (0, 1)$, which of the following is true ?

- (a) $e^x < 1 + x$ (b) $\log_e(1 + x) < x$
(c) $\sin x > x$ (d) $\log_e x > x$

[IIT 2000]

(64) The function $f(x) = \sin^4 x + \cos^4 x$ increases if

- (a) $0 < x < \frac{\pi}{8}$ (b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
(c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

[IIT 1999]

(65) The function $f(x) = [x]^2 - [x^2]$ where $[y]$ is the greatest integer less than or equal to y , is discontinuous at

- (a) all integers (b) all integers except 0 and 1
(c) all integers except 0 (d) all integers except 1

[IIT 1999]

(66) The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ is NOT differentiable at

- (a) -1 (b) 0 (c) 1 (d) 2

[IIT 1999]

(67) $\lim_{x \rightarrow 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2} =$

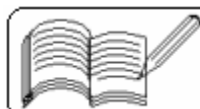
- (a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

[IIT 1999]

(68) The function $f(x) = \int_{-1}^x t(e^t - 1)(t - 1)(t - 2)^3(t - 3)^5 dt$ has a local minimum at $x =$

- (a) 0 (b) 1 (c) 2 (d) 3

[IIT 1999]



(69) $\lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$

- (a) exists and is equal to $\sqrt{2}$ (b) exists and is equal to $-\sqrt{2}$
(c) does not exist because $x - 1 \rightarrow 0$
(d) does not exist because left hand limit \neq right hand limit

[IIT 1998]

(70) If $\int_0^x f(t)dt = x + \int_1^x tf(t)dt$, then the value of $f(1)$ is

- (a) $\frac{1}{2}$ (b) 0 (c) 1 (d) $-\frac{1}{2}$

[IIT 1998]

(71) Let $h(x) = \min [x, x^2]$, for every real number x , then

- (a) h is continuous for all x (b) h is differentiable for all x
(c) $h'(x) = 1$ for all $x > 1$ (d) h is not differentiable at two values of x

[IIT 1998]

(72) If $h(x) = f(x) - [f(x)]^2$ for every real number x , then

- (a) h is increasing whenever f is increasing
(b) h is increasing whenever f is decreasing
(c) h is decreasing whenever f is decreasing
(d) nothing can be said in general

[IIT 1998]

(73) If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \leq 1$, then in this interval

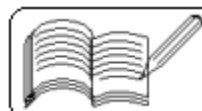
- (a) both $f(x)$ and $g(x)$ are increasing functions
(b) both $f(x)$ and $g(x)$ are decreasing functions
(c) $f(x)$ is an increasing function
(d) $g(x)$ is an increasing function

[IIT 1997]

(74) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ equals

- (a) $1 + \sqrt{5}$ (b) $-1 + \sqrt{5}$ (c) $-1 + \sqrt{2}$ (d) $1 + \sqrt{2}$

[IIT 1997]



(75) If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, (p is a constant), then $\frac{d^3}{dx^3} [f(x)]$ at $x = 0$ is

(a) p (b) $p + p^2$ (c) $p + p^3$ (d) independent of p [IIT 1997]

(76) The function $f(x) = [x] \cos \left[\frac{2x-1}{2} \right] \pi$, where $[.]$ denotes the greatest integer function, is discontinuous at

(a) all x (b) all integer points
(c) no x (d) x which is not an integer [IIT 1995]

(77) If $f(x)$ is defined and continuous for all $x > 0$ and satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and $f(e) = 1$, then

(a) $f(x)$ is bounded (b) $\left(\frac{1}{x}\right) \rightarrow 0$ as $x \rightarrow 0$
(c) $xf(x) \rightarrow 1$ as $x \rightarrow 0$ (d) $f(x) = \log x$ [IIT 1995]

(78) On the interval $[0, 1]$, the function $x^{25}(1-x)^{75}$ attains maximum value at the point

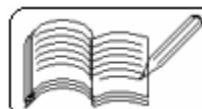
(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ [IIT 1995]

(79) The function $f(x) = |px - q| + r|x|$, $x \in (-\infty, \infty)$ where $p > 0, q > 0, r > 0$, assumes its minimum value only at one point if

(a) $p \neq q$ (b) $r \neq q$ (c) $r \neq p$ (d) $p = q = r$ [IIT 1995]

(80) The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$ is

(a) increasing on $[0, \infty)$ (b) decreasing on $[0, \infty)$
(c) increasing on $\left[0, \frac{\pi}{e}\right]$ and decreasing on $\left[\frac{\pi}{e}, \infty\right)$
(d) decreasing on $\left[0, \frac{\pi}{e}\right]$ and increasing on $\left[\frac{\pi}{e}, \infty\right)$ [IIT 1995]



(81) The function $f(x) = \max \{(1 - x), (1 + x), 2\}$, $x \in (-\infty, \infty)$, is

- (a) continuous at all points (b) differentiable at all points
(c) differentiable at all points except at $x = 1$ and $x = -1$
(d) continuous at all points except at $x = 1$ and $x = -1$

[IIT 1995]

(82) Let $[.]$ denote the greatest integer function and $f(x) = [\tan^2 x]$. Then,

- (a) $\lim_{x \rightarrow 0} f(x)$ does not exist (b) $f(x)$ is continuous at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$ (d) $f'(0) = 1$

[IIT 1993]

(83) If $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \leq x \leq 2 \\ 37 - x, & 2 < x \leq 3 \end{cases}$, then

- (a) $f(x)$ is increasing on $[-1, 2]$ (b) $f(x)$ is continuous on $[-1, 3]$
(c) $f(x)$ is maximum at $x = 2$ (d) $f'(2)$ does not exist

[IIT 1993]

(84) The value of $\lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ is

- (a) 1 (b) -1 (c) 0 (d) none of these

[IIT 1991]

(85) The following functions are continuous on $(0, \pi)$.

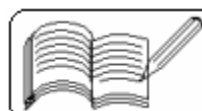
- (a) $\tan x$ (b) $\int_0^{\pi} t \sin \frac{1}{t} dt$
(c) $1, \quad 0 < x \leq \frac{3\pi}{4}$ (d) $x \sin x, \quad 0 < x \leq \frac{\pi}{2}$
 $2 \sin \frac{2x}{9}, \quad \frac{3\pi}{4} < x \leq \pi$ $\frac{\pi}{2} \sin(\pi + x), \quad \frac{\pi}{2} < x < \pi$

[IIT 1991]

(86) If $f(x) = \frac{x}{2} - 1$, then, on the interval $[0, \pi]$, $\tan[f(x)]$ and

- (a) $\frac{1}{f(x)}$ are both continuous (b) $\frac{1}{f(x)}$ are both discontinuous
(c) $f^{-1}(x)$ are both continuous (d) $f^{-1}(x)$ are both discontinuous

[IIT 1989]



(87) If $y^2 = P(x)$, a polynomial of degree 3, then $2 \frac{d}{dx} \left(y^3 \frac{d^2y}{dx^2} \right)$ equals

- (a) $P'''(x) + P'(x)$ (b) $P''(x)P'''(x)$
(c) $P(x)P'''(x)$ (d) a constant

[IIT 1988]

(88) The function $f(x) = \begin{cases} x - 3 & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & x < 1 \end{cases}$ is

- (a) continuous at $x = 1$ (b) differentiable at $x = 1$
(c) continuous at $x = 3$ (d) differentiable at $x = 3$

[IIT 1988]

(89) The set of all points where the function $f(x) = \frac{x}{1 + |x|}$ is differentiable is

- (a) $(-\infty, \infty)$ (b) $(0, \infty)$ (c) $(-\infty, 0) \cup (0, \infty)$
(d) $(0, \infty)$ (e) none of these

[IIT 1987]

(90) Let f and g be increasing and decreasing functions respectively from $(0, \infty)$ to $(0, \infty)$. Let $h(x) = f[g(x)]$. If $h(0) = 0$, $h(x) - h(1)$ is

- (a) always zero (b) always negative (c) always positive
(d) strictly increasing (e) none of these

[IIT 1987]

(91) Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \dots + a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \dots < a_n$. The function $P(x)$ has

- (a) neither a maximum nor a minimum (b) only one maximum
(c) only one minimum (d) only one maximum and only one minimum
(e) none of these

[IIT 1986]

(92) The function $f(x) = 1 + |\sin x|$ is

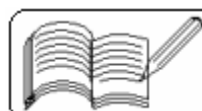
- (a) continuous nowhere (b) continuous everywhere (c) differentiable
(d) not differentiable at $x = 0$ (e) not differentiable at infinite number of points

[IIT 1986]

(93) Let $[x]$ denote the greatest integer less than or equal to x . If $f(x) = [x \sin \pi x]$, then $f(x)$ is

- (a) continuous at $x = 0$ (b) continuous in $(-1, 0)$ (c) differentiable at $x = 1$
(d) differentiable in $(-1, 1)$ (e) none of these

[IIT 1986]



(94) If $f(x) = \frac{\sin[x]}{[x]}$, $[x] \neq 0$
 $= 0$, $[x] = 0$,
where $[x]$ denotes the greatest integer less than or equal to x , then $\lim_{x \rightarrow 0} f(x)$ equals

(a) 1 (b) 0 (c) -1 (d) none of these [IIT 1985]

(95) If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then

(a) $f(x)$ is continuous but not differentiable at $x = 0$
(b) $f(x)$ is differentiable at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$ (d) none of these [IIT 1985]

(96) $\lim_{n \rightarrow \infty} \left\{ \frac{1}{1-n^2} + \frac{2}{1-n^2} + \dots + \frac{n}{1-n^2} \right\}$ is equal to

(a) 0 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) none of these [IIT 1984]

(97) If $x + |y| = 2y$, then y as a function of x is

(a) defined for all real x (b) continuous at $x = 0$
(c) differentiable for all x (d) such that $\frac{dy}{dx} = \frac{1}{3}$ for $x < 0$ [IIT 1984]

(98) If $G(x) = -\sqrt{25-x^2}$, then $\lim_{x \rightarrow 1} \frac{G(x) - G(1)}{x - 1}$ has the value

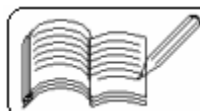
(a) $\frac{1}{24}$ (b) $\frac{1}{5}$ (c) $-\sqrt{24}$ (d) none of these [IIT 1983]

(99) If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$, $g'(a) = 2$, then the value of
 $\lim_{x \rightarrow a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is

(a) -5 (b) $\frac{1}{5}$ (c) 5 (d) none of these [IIT 1983]

(100) The function $f(x) = \frac{\ln(1+ax) - \ln(1-bx)}{x}$ is not defined at $x = 0$. The value which should be assigned to f at $x = 0$, so that it is continuous at $x = 0$, is

(a) $a - b$ (b) $a + b$ (c) $\ln a + \ln b$ (d) none of these [IIT 1983]



(101) The normal to the curve $x = a (\cos \theta + \theta \sin \theta)$, $y = a (\sin \theta - \theta \cos \theta)$ at any point ' θ ' is such that

- (a) it makes a constant angle with the X-axis (b) it passes through the origin
(c) it is at a constant distance from the origin (d) none of these

[IIT 1983]

(102) If $y = a \ln x + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then

- (a) $a = 2, b = -1$ (b) $a = 2, b = -\frac{1}{2}$
(c) $a = -2, b = \frac{1}{2}$ (d) none of these

[IIT 1983]

(103) There exists a function $f(x)$ satisfying $f(0) = 1$, $f'(0) = -1$, $f(x) > 0$ for all x and

- (a) $f''(x) > 0$ for all x (b) $-1 < f''(x) < 0$ for all x
(c) $-2 \leq f''(x) \leq -1$ for all x (d) $f''(x) < -2$ for all x

[IIT 1982]

(104) For a real number y , let $[y]$ denote the greatest integer less than or equal to y . Then the function $f(x) = \frac{\tan[\pi(x - \pi)]}{1 + [x]^2}$ is

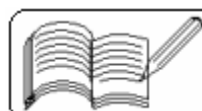
- (a) discontinuous at some x
(b) continuous at all x , but the derivative $f'(x)$ does not exist for some x
(c) $f'(x)$ exists for all x , but the derivative $f''(x)$ does not exist for some x
(d) $f''(x)$ exists for all x

[IIT 1981]

(105) If $f(x) = \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$, then $\lim_{x \rightarrow \infty} f(x)$ is

- (a) 0 (b) ∞ (c) 1 (d) none of these

[IIT 1979]



Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	d	c	a	c	a	b	b	d	b	b	c	c	d	b	a	a	a	c	c
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	d	c	d	c	c	d	b	d	a	a	d	d	d	d	a	a	c	a	b
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
b	d	b	a	c	d	a	d	c	c	d	d	a	a	b	a	d	b	c	d
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
d	d	b	b	b	d	c	b,d	d	a	a,c,d	a,c	c	b	d	b	d	b	c	b
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96				
a,c	b	a,b,c,d	d	b,c	b	c	a,b,c	a	a	c	b,d,e	a,d	d	a	b				
97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113			
a,b,d	d	c	b	c	b	a	d	c											

