07 - SEQUENCES AND SERIES

(Answers at he end of all questions)

(1) If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$, then x, y, z are in

(a) G.P. (b) A.P. (c) Arithmetic-Geometric Progression (d) H.P. [AIEEE 2005]

(2) The sum of the series $1 + \frac{1}{4 \cdot 2!} + \frac{1}{16 \cdot 4!} + \frac{1}{64 \cdot 6!} + \ldots\ldots\ldots$ ad inf. is

(a) $\frac{e - 1}{\sqrt{e}}$ (b) $\frac{e + 1}{\sqrt{e}}$ (c) $\frac{e - 1}{2\sqrt{e}}$ (d) $\frac{e + 1}{2\sqrt{e}}$ [AIEEE 2005]

(3) If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n} =$

(a) $\frac{1}{2} n$ (b) $\frac{1}{2} n - 1$ (c) $n - 1$ (d) $\frac{2n - 1}{2}$ [AIEEE 2004]

(4) Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then

(a) 0 (b) 1 (c) $\frac{1}{mn}$ (d) $\frac{1}{m} + \frac{1}{n}$ [AIEEE 2004]

(5) The sum of the first n terms of he series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \ldots\ldots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd, the sum is

(a) $\frac{3n(n+1)}{2}$ (b) $\frac{n^2(n+1)}{2}$

(c) $\frac{n(n+1)^2}{4}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$ [AIEEE 2004]

(6) The sum of the series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots$ is

(a) $\frac{e^2 - 1}{2}$ (b) $\frac{(e - 1)^2}{2e}$ (c) $\frac{e^2 - 1}{2e}$ (d) $\frac{e^2 - 2}{e}$ [AIEEE 2004]

(7) The sum of the series $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots\ldots \infty$ is

(a) $\log_e 2$ (b) $2\log_e 2$ (c) $\log_e 2 - 1$ (d) $\log_e \frac{4}{e}$ [AIEEE 2003]

(8) If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in

(a) A. P. (b) G. P. (c) H. P. (d) A. G. P. [AIEEE 2003]

(9) The value of 1. 2. 3 + 2. 3. 4 + 3. 4. 5 + ... + n terms is

(a) $\frac{n(n+1)(n+2)(n+3)}{12}$ (b) $\frac{n(n+1)(n+2)(n+3)}{3}$

(c) $\frac{n(n+1)(n+2)(n+3)}{4}$ (d) $\frac{(n+2)(n+3)(n+4)}{6}$ [AIEEE 2002]

(10) If the third term of an A. P. is 7 and its 7th term is 2 more than three times of its third term, then the sum of its first 20 terms is

(a) 228 (b) 74 (c) 740 (4) 1090 [AIEEE 2002]

(11) An infinite G. P. has first term ‘ x ’ and sum 5, then

(a) $x \geq 10$ (b) $0 < x < 10$ (c) $x < -10$ (d) $-10 < x < 0$ { IIT 2004 }

(12) If $a_1, a_2, \ldots\ldots a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + \ldots + a_{n-1} + 2a_n$ is

(a) $n(2c)^{1/n}$ (b) $(n+1)c^{1/n}$ (c) $2nc^{1/n}$ (d) $(n+1)(2c)^{1/n}$ [IIT 2002]

(13) Suppose a, b, c are in A. P. and a^2, b^2, c^2, are in G. P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$ (c) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (d) $\frac{1}{2} - \frac{1}{\sqrt{2}}$ [IIT 2002]

(14) If the sum of the first 2n terms of the A. P. 2, 5, 8, ….. , is equal to the sum of the first n terms of the A. P. 57, 59, 61, ….. , then n equals

(a) 10 (b) 12 (c) 11 (d) 13 [IIT 2001]

(15) If the positive numbers a, b, c, d are in A. P., then abc, abd, acd, bcd are

(a) not in A. P. / G. P. / H. P. (b) in A. P. (c) in G. P. (d) in H. P. [IIT 2001]

(16) If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b) (c + d)$ satisfies the relation

(a) $0 \le M \le 1$ (b) $1 \le M \le 2$ (c) $2 \le M \le 3$ (d) $3 \le M \le 4$ [IIT 2000]

(17) Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then a and r are

(a) $\frac{4}{7}, \frac{3}{7}$ (b) $2, \frac{3}{8}$ (c) $\frac{3}{2}, \frac{1}{2}$ (d) $3, \frac{1}{4}$ [IIT 2000]

(18) Let $a_1, a_2, \ldots, a_{10}$ be in A. P. and $h_1, h_2, \ldots, h_{10}$ be in H. P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is

(a) 2 (b) 3 (c) 5 (d) 6 [IIT 1999]

(19) If for a positive integer n, $a (n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots.. + \frac{1}{2^n - 1}$, then

(a) $a (100) \le 100$ (b) $a (100) > 100$
(c) $a (200) \le 100$ (d) $a (200) > 100$ [IIT 1999]

(20) Let T_r be the rth term of an A. P., for r = 1, 2, 3, … If for some positive integers m, n, we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals

(a) $\frac{1}{mn}$ (b) $\frac{1}{m} + \frac{1}{n}$ (c) 1 (d) 0 [IIT 1998]

(21) If $x > 1$, $y > 1$, $z > 1$ are in G. P., then $\frac{1}{1 + \ln x}$, $\frac{1}{1 + \ln y}$, $\frac{1}{1 + \ln z}$ are in

(a) A. P. (b) H. P. (c) G. P. (d) None of these [IIT 1998]

(22) If $n > 1$ is a positive integer, then the largest integer m such that $(n^m + 1)$ divides $(1 + n + n^2 + \ldots + n^{127})$ is

(a) 127 (b) 63 (c) 64 (d) 32 [IIT 1995]

(23) The product of n positive numbers is unity. Then their sum is

(a) a positive integer (b) divisible by n

(c) equal to $n + \frac{1}{n}$ (d) never less than n [IIT 1991]

(24) The sum of n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots$ is equal to

(a) $2^n - n - 1$ (b) $1 - 2^{-n}$ (c) $n + 2^{-n} + 1$ (d) $2^n - 1$ [IIT 1988]

(25) If the first and the (2n - 1)th terms of an A. P., G. P. and H. P. are equal and their nth terms are a, b, c respectively, then

(a) $a = b = c$ (b) $a \geq b \geq c$ (c) $a + c = b$ (d) $ac - b^2 = 0$ [IIT 1988]

(26) If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2) p^2 - 2 (ab + bc + cd) p + (b^2 + c^2 + d^2) \leq 0$, then a, b, c and d

(a) are in A. P. (b) are in G. P. (c) are in H. P. (d) satisfy $ab = cd$ [IIT 1987]

(27) If a, b, c are in G. P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}$, $\frac{e}{b}$, $\frac{f}{c}$ are in

(a) AP (b) GP (c) HP (d) none of these [IIT 1985]

(28) The third term of a geometric progression is 4. The product of the first five terms is

(a) 4^3 (b) 4^5 (c) 4^4 (d) none of these [IIT 1982]

(29) If $x_1, x_2, \ldots, x_n$ are any real numbers and n is any positive integer, then

(a) $n \sum_{i=1}^{n} x_i^2 < \left(\sum_{i=1}^{n} x_i \right)^2$ (b) $\sum_{i=1}^{n} x_i^2 \geq \left(\sum_{i=1}^{n} x_i \right)^2$

(c) $\sum_{i=1}^{n} x_i^2 \geq n \left(\sum_{i=1}^{n} x_i \right)^2$ (d) none of these [IIT 1982]

(30) If x, y and z are the p th, q th and r th terms respectively of an A. P. and also of a G. P., then $x^{y-z} \, y^{z-x} \, z^{x-y}$ is equal to

(a) xyz (b) 0 (c) 1 (d) none of these [IIT 1979]

(31) $\frac{1}{1 + \sqrt{x}}$, $\frac{1}{1 - x}$ and $\frac{1}{1 - \sqrt{x}}$ are consecutive terms of a series in

(a) H. P. (b) G. P. (c) A. P. (d) A. P., G. P.

(32) If $S_n = nP + \frac{1}{2} n(n-1)Q$, where S_n denotes the sum of the first n terms of an A. P., then the common difference is

(a) P + Q (b) 2P + 3Q (c) 2Q (d) Q

(33) If $S_n = n^3 + n^2 + n + 1$, where S_n denotes the sum of the first n terms of a series and $t_m = 291$, then m =

(a) 10 (b) 11 (c) 12 (d) 13

(34) If the first term minus third term of a G. P. = 768 and the third term minus seventh term of the same G. P. = 240, then the product of first 21 terms =

(a) 1 (b) 2 (c) 3 (d) 4

07 - SEQUENCES AND SERIES

(Answers at he end of all questions)

(35) If the sequence $a_1, a_2, a_3, \ldots a_n$ form an A. P., then $a_1^2 - a_2^2 + a_3^2 - \ldots + a_{2n-1}^2 - a_{2n}^2 =$

(a) $\frac{n}{2n - 1}(a_1^2 - a_{2n}^2)$ (b) $\frac{2n}{n - 1}(a_{2n}^2 - a_1^2)$

(c) $\frac{n}{n + 1}(a_1^2 + a_{2n}^2)$ (d) None of these

(36) If T_r denotes rth term of an H. P. and $\frac{T_1 - T_4}{T_6 - T_9} = 7$, then $\frac{T_2 - T_5}{T_{11} - T_8} =$

(a) 5 (b) 6 (c) 7 (d) 8

(37) The sum of any ten positive real numbers multiplied by the sum of their reciprocals is

(a) ≥ 10 (b) ≥ 50 (c) ≥ 100 (d) ≥ 200

(38) If S_n denotes the sum of first n terms of an A. P. and $S_{2n} = 3S_n$, then the ratio $\frac{S_{3n}}{S_n}$ is equal to

(a) 4 (b) 6 (c) 8 (d) 10

(39) If a, b, c are three unequal positive quantities in H. P., then

(a) $a^{10} + c^{10} < 2b^{10}$ (b) $a^{20} + c^{20} < 2b^{20}$
(c) $a^3 + c^3 < 2b^3$ (d) none of these

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	d	a	a	b	b	d	c	c	c	b	a	d	c	d	a	d	d	a,d	c

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	c	d	c	b,d	b	a	b	d	c	c	d	a	a	a	b	c	b	d	