(1) Find the equations in vector as well as Cartesian form of a sphere with centre (2, 3, 4) and passing through (4, 4, 4).

[Ans: $|\bar{r}|^2 - 2\,(\,2,\,3,\,4\,)\cdot\bar{r} + 24 = 0,\quad x^2 + y^2 + z^2 - 4x - 6y - 8z + 24 = 0$]

(2) Find the centre and length of radius of the sphere $\bar{r}^2 - \bar{r}\cdot(\,3,\,1,\,-1\,) + 2 = 0.$

$$\left[\; \text{Ans}:\; \left(\frac{3}{2},\,\frac{1}{2},\,-\frac{1}{2}\right),\;\; \frac{\sqrt{3}}{2}\; \right]$$

(3) Find the centre and radius of the sphere represented by $2x^2 + 2y^2 + 2z^2 - 4x + 1 = 0.$

$$\left[\; \text{Ans}:\; (1,\,0,\,0),\;\; \frac{1}{\sqrt{2}}\; \right]$$

(4) Obtain the equation, the centre and length of a radius of the sphere through (0, 0, 0), (a, 0, 0), (0, b, 0) and (0, 0, c).

$$\left[\; \text{Ans}:\; x^2 + y^2 + z^2 - ax - by - cz = 0,\;\; \frac{1}{2}\sqrt{a^2 + b^2 + c^2}\; \right]$$

(5) If the diameter of the sphere has end-points (1, 2, -1) and (2, 1, 0), obtain the equation of the sphere.
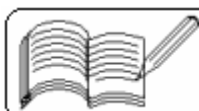
[Ans: $x^2 + y^2 + z^2 - 3x - 3y + z + 4 = 0$]

(6) Derive the equation of the sphere through (0, 0, 0), (- a, b, c), (a, - b, c) and (a, b, - c).

$$\left[\; \text{Ans}:\; x^2 + y^2 + z^2 - (a^2 + b^2 + c^2)\left(\frac{x}{a} + \frac{y}{b} + \frac{z}{c}\right)\; \right]$$

(7) Find the centre and length of radius of a sphere represented by the equation
$\bar{r}^2 - \bar{r}\cdot(\,6,\,12,\,14\,) + 30 = 0.$

[Ans: (3, 6, 7), 8]

(8) Verify whether or not the following equations represent proper sphere. If they do, then obtain their centre and radius.

(a) $x^2 + y^2 + z^2 = 2ax$,

(b) $5x^2 + 5y^2 + 5z^2 - 5x - 10y + 15z + 21 = 0$.

[Ans: (a) (a, 0, 0), radius $|a|$ (b) does not represent sphere]

(9) If a plane through (a, b, c) intersects the co-ordinate axes in A, B and C, then show that the centre of the sphere OABC is on $\dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 2$, where O is the origin.

(10) Derive the equation of the sphere with the line-segment joining the points (2, - 1, 4) and (-2, 2, - 2) as its diameter. Find the area of the circle in which the sphere is intersected by the plane $2x + y - z = 3$.

$$\left[\text{Ans}: \; x^2 + y^2 + z^2 - y - 2z - 14 = 0, \quad \frac{317}{24}\pi \right]$$

(11) Find the equation of the sphere having the circle given by $x + y + z = 3$ and $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$ as a great circle.

[Ans: $x^2 + y^2 + z^2 - 4x + 6y - 8z + 4 = 0$]

(12) Find the equations of the spheres passing through the circle given by the equations $x^2 + y^2 + z^2 - 6x - 2z + 5 = 0$ and $y = 0$ and touching the plane $3y + 4z + 5 = 0$.
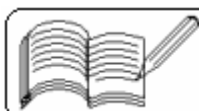
$$\left[\text{Ans}: \; x^2 + y^2 + z^2 - 6x - \frac{11}{4}y - 2z + 5 = 0, \quad x^2 + y^2 + z^2 - 6x - 4y - 2z + 5 = 0 \right]$$

(13) Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ at the point (1, 2, - 2) and passes through the origin.

[Ans: $4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$]

(14) Find the shortest and the longest distance of the sphere $x^2 + y^2 + z^2 = 24$ from the point (1, 2, - 1).

[Ans: $\sqrt{6}$, $3\sqrt{6}$]

(15) A sphere of constant radius r passes through the origin O and cuts the axes in A, B and C. Prove that the locus of the foot of the perpendicular from O on the plane ABC is given by $(x^2 + y^2 + z^2)^2 (x^{-2} + y^{-2} + z^{-2}) = 4r^2$.

(16) In the above example, prove that the locus of the centroid of the triangle ABC is given by $9(x^2 + y^2 + z^2) = 4r^2$.

(17) Tangent plane at any point of the sphere $x^2 + y^2 + z^2 = a^2$ meets the co-ordinate axes at A, B and C. Show that the locus of the point of intersection of the planes drawn parallel to the co-ordinate planes through A, B, C is the surface whose equation is given by $x^{-2} + y^{-2} + z^{-2} = a^{-2}$.

(18) Derive the equations of all the spheres touching the co-ordinate planes and the plane x + 2y + 2z = 40.

[Ans: (i) $x^2 + y^2 + z^2 - 10(x + y + z) + 50 = 0,$
(ii) $x^2 + y^2 + z^2 - 40(x + y + z) + 800 = 0,$
(iii) $9(x^2 + y^2 + z^2) + 120(x - y - z) + 800 = 0,$
(iv) $x^2 + y^2 + z^2 - 20(x - y + z) + 200 = 0,$
(v) $x^2 + y^2 + z^2 - 20(x + y - z) + 200 = 0,$
(vi) $x^2 + y^2 + z^2 + 40(x + y - z) + 800 = 0,$
(vii) $x^2 + y^2 + z^2 + 40(x - y + z) + 800 = 0. $]

(19) The sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ and a point (2, 4, 5) are given. Prove that the volume of the double cone formed whose circular base is the circle of intersection of the given sphere and plane of contact from the given point and vertices are the centre of the circle and the given point is $\dfrac{20\pi}{3}$.

(20) Find the equation of the sphere of minimum volume touching the spheres $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ and $x^2 + y^2 + z^2 - 6x - 12y - 14z + 90 = 0$.

[Ans: $x^2 + y^2 + z^2 - 4x - 8y - 10z + 44 = 0$]