(1) Obtain the vector and Cartesian equations of the plane through A (1, 2, 3), B (2, 1, 0) and C (3, 3, - 1).

[Ans: $\bar{r} = (1, 2, 3) + m(1, -1, -3) + n(2, 1, -4)$, $7x - 2y + 3z = 12$]

(2) A plane intersects X-, Y- and Z-axes at A, B and C respectively. The centroid of triangle ABC is (p, q, r). Derive the equation of the plane.

$$\left[\text{Ans : } \frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3 \right]$$

(3) Obtain the foot of perpendicular from the point (1, 2, 3) on the plane $x - 2y + 2z = 5$ and the distance of the point from the plane.

$$\left[\text{Ans : } \left(\frac{11}{9}, \frac{14}{9}, \frac{31}{9} \right), \frac{2}{3} \right]$$

(4) Find the image of (1, 3, 4) relative to the plane $2x - y + z + 3 = 0$.

[Ans: (- 3, 5, 2)]

(5) Find the common section of $x + 2y - 3z = 6$ and $2x - y + z = 7$.

$$\left[\text{Ans : } \frac{x - 4}{1} = \frac{y - 1}{7} = \frac{z}{5} \right]$$

(6) Obtain the equation of the plane through (1, 3, 5) perpendicular to the intersection of $3x + y - z = 0$ and $x + 2y + 3z = 5$.

[Ans: $x - 2y + z = 0$]

(7) Obtain the equation of the plane containing $\bar{r} = (1, 1, 1) + m(2, 1, 2)$ $k \in R$ and (1, - 1, 2).

[Ans: $5x - 2y - 4z + 1 = 0$]

(8) Obtain the equation of a plane passing through $\frac{x - 4}{1} = \frac{y + 3}{-4} = \frac{z + 1}{7}$ and $\frac{x - 1}{2} = \frac{y + 1}{-3} = \frac{z + 10}{8}$.

[Ans: $11x - 6y - 5z = 67$]

(9) Find the length, the foot and the equation of perpendicular from $(2, -1, 2)$ to the plane $2x - 3y + 4z = 44$.

[Ans: $\sqrt{29}$, $(4, -4, 6)$, $\bar{r} = (2, -1, 2) + k (2, -3, 4)$, $k \in R$]

(10) Find the (perpendicular) distance between the planes $3x - 2y + z = 1$ and $6x - 4y + 2z = 5$.

$\left[\text{Ans : } \frac{3}{2\sqrt{14}} \right]$

(11) Find the equation of the plane through $(1, 1, 1)$ and the line of intersection of planes $x + 2y + 3z = 4$ and $4x + 3y + z + 1 = 0$.

[Ans: $x + 12y + 25z = 38$]

(12) Find the equation of the plane through $(1, 2, 3)$ and $(3, -1, 2)$ perpendicular to the plane $x + 3y + 2z = 7$.

[Ans: $3x + 5y - 9z + 14 = 0$]

(13) Two systems of rectangular axes have the same origin. If the plane cuts them at a, b, c and a_1, b_1, c_1 on the co-ordinate axes respectively from the origin, then show that $a^{-2} + b^{-2} + c^{-2} = a_1^{-2} + b_1^{-2} + c_1^{-2}$.

(14) Find the equations of the planes bisecting the angle between the planes $x + 2y + 2z = 9$ and $4x - 3y + 12z + 12 = 0$.

[Ans: $x + 35y - 10z - 153 = 0$, $25x + 17y + 62z - 81 = 0$]

(15) Find the equations of the two planes through the points $(0, 4, -3)$ and $(6, -4, 3)$ other than the plane through the origin which cut off intercepts from the axes whose sum is zero.

[Ans: $2x - 3y - 6z = 6$, $6x + 3y - 2z = 18$]

(16) Find the equation of the plane passing through the line of intersection of the planes $2x + y + 3z - 4 = 0$ and $4x - y + 5z - 7 = 0$ and perpendicular to yz-plane.

[Ans: $3y + z = 1$]

(17) A plane contains the points $A (-4, 9, -9)$ and $B (5, -9, 6)$ and is perpendicular to the line which joins B and $C (4, -6, k)$. Evaluate k and find the equation of the plane.

[Ans: $k = 10.2$, $5x - 15y - 21z = 34$]

(18) Prove that the equation of the plane which bisects the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) at right angles is

$$\sum (x_1 - x_2) \left(x - \frac{x_1 + x_2}{2} \right) = 0 .$$

(19) Find the equation of the plane passing through the line of intersection of the planes $3x + 3y + 2 = 23$ and $x + 3y + 6z = 35$ which is at the shortest distance from the origin.

[Ans: $2x + 3y + 4z = 29$]

(20) $P_1 : x + 2y + 2z + 1 = 0$ and $P_2 : 2x + 2y + z + 2 = 0$ are the equations of two planes having angle θ between them. Find the equation of the plane other than P_1 which makes the same angle θ with the plane P_2 .

[Ans: $23x + 14y - 2z + 23 = 0$]

(21) A plane is drawn through the line $x + y = 1$, $z = 0$ to make an angle $\sin^{-1} (1 / 3)$ with the plane $x + y + z = 0$. Prove that two such planes can be drawn and find their equations. Also, prove that the angle between the planes is $\cos^{-1} (7 / 9)$.

[Ans: $x + y - z = 1$ and $x + y - 5z = 1$]