

- (1) Obtain the equations of the line through $A(1, 2, 1)$ and $B(2, 3, -1)$ in vector and Cartesian forms.

$$\left[\text{Ans: } \bar{r} = (1, 2, 1) + k(1, 1, -2), k \in \mathbb{R}, \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-1}{-2} \right]$$

- (2) Find the angle between $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z+1}{-2}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z+3}{3}$.

$$\left[\text{Ans: } \cos^{-1} \sqrt{\frac{3}{154}} \right]$$

- (3) Prove that the lines $\bar{r} = (1, 2, 6) + k(1, 3, 5)$, $k \in \mathbb{R}$ and $\bar{r} = (-1, 3, 5) + k(2, 1, 1)$, $k \in \mathbb{R}$ are non-coplanar.

- (4) Obtain the perpendicular distance of the line $\bar{r} = (2, 1, 5) + k(1, 0, 1)$, $k \in \mathbb{R}$ from $P(1, 2, 1)$.

$$\left[\text{Ans: } \sqrt{\frac{11}{2}} \right]$$

- (5) Show that $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{5}$ and $\frac{x}{3} = \frac{y-1}{2} = \frac{z-1}{1}$ are skew lines. Find the shortest distance between the.

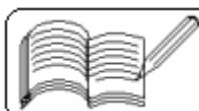
$$\left[\text{Ans: } \frac{2}{\sqrt{3}} \right]$$

- (6) Obtain the co-ordinates of the foot of perpendicular from $(2, 4, -1)$ on $\bar{r} = (-5, -3, 6) + k(1, 4, -9)$, $k \in \mathbb{R}$ and find the distance of the point from the line.

$$[\text{Ans: } (-4, 1, -3), 7]$$

- (7) Obtain the vector and Cartesian forms of equations of the line through $A(1, 2, 3)$ in the direction $(-1, 1, 2)$ and find the co-ordinates of the points on the line at a distance $\sqrt{6}$ from A.

$$\left[\text{Ans: } \bar{r} = (1, 2, 3) + k(-1, 1, 2), \frac{x-1}{-1} = \frac{y-2}{1} = \frac{z-3}{2}, k \in \mathbb{R}, (0, 3, 5), (2, 1, 1) \right]$$



- (8) Determine which among the following pairs of lines are coincident, parallel, intersecting or skew.
- (a) $\{(1, 3, 5) + k(-1, 2, 3) \mid k \in \mathbb{R}\}$ and $\{(1, 3, 1) + k(1, -2, -3) \mid k \in \mathbb{R}\}$
- (b) $\bar{r} = (2, 1, 3) + k(1, -1, 1), k \in \mathbb{R}$ and $\bar{r} = (3, 0, 4) + k(-1, 1, -1), k \in \mathbb{R}$
- (c) $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{5} = \frac{z+2}{2}$
- (d) $\{(k+3, -k-1, k+1) \mid k \in \mathbb{R}\}$ and $\{(2k, 0, 3k-3) \mid k \in \mathbb{R}\}$.
- [Ans: (a) parallel (b) coincident (c) skew (d) intersecting]

- (9) Find the (perpendicular) distance between the parallel lines $x - 1 = y + 2 = z - 3$ and $x - 2 = y + 3 = z - 5$.

$$\left[\text{Ans: } \sqrt{\frac{14}{3}} \right]$$

- (10) Find the co-ordinates of points on the line $\bar{r} = (1, 2, 1) + k(-1, -2, 1), k \in \mathbb{R}$ distant $\sqrt{6}$ units from $(2, 4, 0)$.

$$[\text{Ans: } (1, 2, 1), (3, 6, -1)]$$

- (11) Find equation of the line through $(3, -1, 11)$ perpendicular to $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

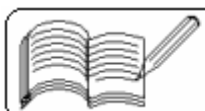
$$[\text{Ans: } \bar{r} = (3, -1, 11) + k(1, -6, 4), k \in \mathbb{R}]$$

- (12) Find the shortest distance between $\bar{r} = (2, 1, -2) + k(3, -5, 2), k \in \mathbb{R}$ and $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$.

$$\left[\text{Ans: } \frac{1}{\sqrt{3}} \right]$$

- (13) Show that the angle between any two diagonals of a cube is $\cos^{-1}(1/3)$.

[Note: This angle is $109^\circ 28'$ which is the angle between any two C - H bonds of methane.]



(14) If a line makes angles α , β , γ and δ with the diagonals of a cube, then show that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma + \sin^2 \delta = \frac{8}{3}$.

(15) If the direction cosines l , m , n of two lines satisfy $l + m + n = 0$ and $l^2 - m^2 + n^2 = 0$, then show that the angle between the two lines is $\frac{\pi}{3}$.

(16) Show that the lines whose direction cosines are given by the equations $l + m + n = 0$, $al^2 + bm^2 + cn^2 = 0$ are
 (i) perpendicular, if $a + b + c = 0$ and
 (ii) parallel, if $a^{-1} + b^{-1} + c^{-1} = 0$.

(17) Show that the lines whose direction cosines are given by the equations $ul + vm + wn = 0$ and $fmn + gn + h/m = 0$ are perpendicular if $\frac{f}{u} + \frac{g}{v} + \frac{h}{w} = 0$.

(18) Prove that the lines $x = ay + b = cz + d$ and $x = \alpha y + \beta = \gamma z + \delta$ are coplanar if $(\gamma - c)(a\beta - b\alpha) = (\alpha - a)(c\delta - d\gamma)$.

(19) Find the equation of the straight line perpendicular to both the lines $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z+2}{3}$ and $\frac{x+2}{2} = \frac{y-5}{-1} = \frac{z+3}{2}$ and passing through their point of intersection.

[Ans: $\vec{r} = (2, 3, 1) + k(7, 4, -5)$, $k \in \mathbb{R}$]

(20) Show that the shortest distances between a diagonal of a rectangular parallelepiped whose edges are a , b , c and the edges not meeting it are

$$\frac{bc}{\sqrt{b^2 + c^2}}, \quad \frac{ca}{\sqrt{c^2 + a^2}} \quad \text{and} \quad \frac{ab}{\sqrt{a^2 + b^2}}.$$

(21) If the line $L: x = ay + b = cz + d$ is perpendicular to the line $M: y = a'x + b = c'z + d$, then prove that

$$cc'(a + a') + aa' = 0.$$

