## Solve all problems vectorially:

( 1 ) Obtain the unit vectors perpendicular to each of $\bar{x} = (1, 2, -1)$ and $\bar{y} = (1, 0, 2)$.

$$\left[ \text{Ans: } \pm \left( \frac{4}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \right) \right]$$

( 2 ) If $\alpha$ is the angle between two unit vectors $\bar{a}$ and $\bar{b}$, then prove that $|\bar{a} - \bar{b} \cos\alpha| = \sin\alpha$.

( 3 ) If a vector $\bar{r}$ makes with X-axis and Y-axis angles of measures $45°$ and $60°$ respectively, then find the measure of the angle which $\bar{r}$ makes with Z-axis.

[ Ans: $60°$ or $120°$ ]

( 4 ) If $\bar{x}$ and $\bar{y}$ are non-collinear vectors of $R^3$, then prove that $\bar{x}$, $\bar{y}$ and $\bar{x} \times \bar{y}$ are non-coplanar.

( 5 ) If the measure of angle between $\bar{x} = \bar{i} + \bar{j}$ and $\bar{y} = t\bar{i} - \bar{j}$ is $\frac{3\pi}{4}$, then find t.

[ Ans: 0 ]

( 6 ) Show that for any $a \in R$, the directions $(2, 3, 5)$ and $(a, a+1, a+2)$ cannot be the same or opposite.

( 7 ) If $\theta$ is a measure of angle between unit vectors $\bar{a}$ and $\bar{b}$, prove that $\sin\frac{\theta}{2} = \frac{1}{2} |\bar{a} - \bar{b}|$.

( 8 ) If $\bar{x}$, $\bar{y}$ and $\bar{z}$ are non-coplanar, prove that $\bar{x} + \bar{y}$, $\bar{y} + \bar{z}$ and $\bar{z} + \bar{x}$ are also non-coplanar.

**( 9 )** Show that the vectors $(1, 2, 1)$, $(1, 1, 4)$ and $(1, 3, -2)$ are coplanar. Also express each of these vectors as a linear combination of the other two.

$$\left[\text{Ans: } (1, 2, 1) = \frac{1}{2}(1, 1, 4) + \frac{1}{2}(1, 3, -2); \quad (1, 1, 4) = 2(1, 2, 1) - (1, 3, -2); \quad (1, 3, -2) = 2(1, 2, 1) - (1, 1, 4)\right]$$

[ Note: These vectors are collinear besides being coplanar. Hence, any vector of $R^3$ which is not collinear with them cannot be expressed as a linear combination of these vectors even if it is coplanar with them. ]

**( 10 )** Show that $(1, 1, 0)$, $(1, 0, 1)$ and $(0, 1, 1)$ are non-coplanar vectors. Also express any vector $(x, y, z)$ of $R^3$ as a linear combination of these vectors.

$$\left[\text{Ans: } (x, y, z) = \frac{x + y - z}{2}(1, 1, 0) + \frac{x - y + z}{2}(1, 0, 1) + \frac{y + z - x}{2}(0, 1, 1)\right]$$

**( 11 )** Prove that an angle in a semi-circle is a right angle.

**( 12 )** Prove that the three altitudes in a triangle are concurrent.

**( 13 )** If $A - P - B$ and $\frac{AP}{PB} = \frac{m}{n}$, then prove that for any point $O$ in space, $n(\overrightarrow{OA}) + m(\overrightarrow{OB}) = (m + n)\overrightarrow{OP}$.

**( 14 )** Prove that $A(1, 5, 6)$, $B(3, 1, 2)$ and $C(4, -1, 0)$ are collinear. Find also the ratio in which $A$ divides $\overline{BC}$ from $B$.

[ Ans: $-2 : 3$ ]

**( 15 )** Find in which ratio and at which point does the XY-plane divide $\overline{AB}$ where $A$ is $(2, -2, 1)$ and $B$ is $(1, 4, -5)$.

$$\left[\text{Ans: } 1 : 5 \text{ from } A \text{ at } \left(\frac{11}{6}, -1, 0\right)\right]$$

( 16 ) If A ( 0, - 1, - 1 ), B ( 16, - 3, - 3 ) and C ( - 8, - 1, - 2 ) are given points, then find the point D ( x, y, z ) in space so that $\overrightarrow{AB} = \overrightarrow{CD}$.

[ Ans: ( 8, - 3, - 4 ) ]

( 17 ) A ( 0, - 1, - 4 ), B ( 1, 2, 3 ) and C ( 5, 4, - 1 ) are given points. If D is the foot of perpendicular from A on $\overline{BC}$, find its position vector.

[ Ans: ( 3, 3, 1 ) ]

( 18 ) If the position vectors A, B, C of triangle ABC are $\bar{a}$, $\bar{b}$, $\bar{c}$ respectively, then show that the area of triangle ABC $= \frac{1}{2} \left| (\bar{a} \times \bar{b}) + (\bar{b} \times \bar{c}) + (\bar{c} \times \bar{a}) \right|$.

( 19 ) Find the volume of a prism having a vertex at origin O and having coterminous edges $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, where A is ( 4, 3, 1 ), B is ( 3, 1, 2 ) and C is ( 5, 2, 1 ).

[ Ans: 10 cubic units ]

( 20 ) Find the volume of tetrahedron having vertices V ( 1, 1, 3 ), A ( 4, 3, 2 ), B ( 5, 2, 7 ) and C ( 6, 4, 8 ).

$\left[ \text{Ans : } \frac{14}{3} \text{ cubic units} \right]$

( 21 ) If the forces of magnitudes $\sqrt{2}$, 2 and $\sqrt{3}$ units are applied to a particle in the directions of vectors ( - 1, 0, 1 ), ( 1, 0, 1 ) and ( 1, 1, - 1 ) respectively, then find the magnitude and direction of the resultant force.

$\left[ \text{Ans : } \sqrt{5}, \left( \cos^{-1}\sqrt{\frac{2}{5}}, \cos^{-1}\sqrt{\frac{1}{5}}, \cos^{-1}\sqrt{\frac{2}{5}} \right) \right]$

( 22 ) A boat is sailing to the east with a speed of $10\sqrt{2}$ km / hr. A man on boat feels that the wind is blowing from the south-east with a speed of 5 km / hr. Find the true velocity of the wind.

$\left[ \text{Ans : } 5\sqrt{5} \text{ km / hr at an angle } \cos^{-1}\frac{3}{\sqrt{10}} \text{ with east towards north} \right]$

**( 23 )** A force of magnitude $2\sqrt{10}$ units is acting on a particle in the direction $3\bar{i} - \bar{j}$ and a force of magnitude $3\sqrt{13}$ units is acting on the same particle in the direction $2\bar{i} + 3\bar{j}$. Under the influence of these forces, the particle is displaced from A ( 1, 2 ) to B ( 6, 4 ). Find the work done. [ Ans: 74 units ]

**( 24 )** Prove that the diagonals of a rhombus bisect each other orthogonally.

**( 25 )** If a pair of medians of a triangle are equal, then show that the triangle is isosceles.

**( 26 )** Show that the perpendicular bisectors of sides of any triangle are concurrent.

**( 27 )** Prove that the diagonals of a rhombus are bisectors of its angles.

**( 28 )** If $\overrightarrow{AD}$ is a bisector of $\angle BAC$ in triangle ABC and if $D \in BC$, then show that $\dfrac{BD}{DC} = \dfrac{AB}{AC}$.

**( 29 )** ABCDEF is a regular hexagon. Prove that $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3\overrightarrow{AD}$.

**( 30 )** Show that centroid and in-centre of an equilateral triangle are the same. Find the incentre of the triangle with vertices ( 6, 4, 6 ), ( 12, 4, 0 ) and ( 4, 2, - 2 ).

$$\left[ \text{Ans : } \left( \frac{22}{3}, \frac{10}{3}, \frac{4}{3} \right) \right]$$

**( 31 )** If A is ( 1, 2, 1 ) and B is ( 4, - 1, 2 ), then find S ( x, y, z ) such that $2\overrightarrow{AB} = \overrightarrow{AS}$.

[ Ans: ( 7, - 4, 3 ) ]

( 32 ) Let A ( 1, 2, - 1 ) and B ( 3, 2, 2 ) be given points. Find in which ratios from A and at which points do the XY-, YZ- and ZX-planes divide $\overline{AB}$.

$$\left[ \text{Ans : } 1 : 2, \ \left( \frac{5}{3}, 2, 0 \right); \ -1 : 3, \ \left( 0, 2, \frac{5}{2} \right); \ \overleftrightarrow{AB} \text{ is parallel to ZX - plane} \right]$$

( 33 ) Show that ( 6, 0, 1 ), ( 8, - 3, 7 ) and ( 2, - 5, 10 ) can be three vertices of some rhombus. Find the co-ordinates of the fourth vertex of this rhombus.

[ Ans: ( 0, - 2, 4 ) ]

( 34 ) Show that ( 1, 2, 4 ), ( - 1, 1, 1 ), ( 6, 3, 8 ) and ( 2, 1, 2 ) are the vertices of a trapezium. Find the area of this trapezium.

$$\left[ \text{Ans : } \frac{3}{2}\sqrt{59} \right]$$

( 35 ) Find the area of the parallelogram ABCD if $\overrightarrow{AC} = \bar{a}$ and $\overrightarrow{BD} = \bar{b}$.

$$\left[ \text{Ans : } \frac{1}{2} \left| \bar{a} \times \bar{b} \right| \right]$$

( 36 ) Find the volume of a prism having a vertex at origin and having edges $\overrightarrow{OA} = 2\bar{i} + \bar{j} + \bar{k}$, $\overrightarrow{OB} = 3\bar{i} - \bar{j} + \bar{k}$ and $\overrightarrow{OC} = -\bar{i} + \bar{j} - \bar{k}$.

[ Ans: 4 cubic units ]

( 37 ) Show that ( 4, 5, 1 ), ( 0, - 1, - 1 ), ( 3, 9, 4 ) and ( - 4, 4, 4 ) cannot be the vertices of any tetrahedron.

( 38 ) Find the volume of the tetrahedron with vertices ( 4, 5, 1 ), ( 0, - 1, - 1 ), ( 3, 9, 4 ) and ( 1, 2, 3 ).

$$\left[ \text{Ans : } \frac{28}{3} \text{ cubic units} \right]$$

**( 39 )** **A mechanical boat is rowing towards the north with speed of 8 km / hr. If wind blows from the east with the speed of 10 km / hr, find the resulting speed of the boat and also the direction of resulting motion of the boat.**

$$\left[ \text{Ans : } 2\sqrt{41} \text{ km / hr at an angle of } \pi - \cos^{-1}\left(\frac{5}{\sqrt{41}}\right) \text{ with east towards north} \right]$$

**( 40 )** **A river flows with a speed of 5 units. A person desires to cross the river in a direction perpendicular to its flow. Find in which direction should he swim if his speed is 8 units.**

$$\left[ \text{Ans : At an angle of } \pi - \cos^{-1}\left(\frac{5}{8}\right) \text{ with the direction of flow of the river} \right]$$

**( 41 )** **If speed of a particle is 5 units towards the east and $\sqrt{8}$ units towards the south-west, then find the resultant speed of the particle and its direction.**

$$\left[ \text{Ans : } \sqrt{13} \text{ units at an angle of } \cos^{-1}\frac{3}{\sqrt{13}} \text{ with east towards south} \right]$$

**( 42 )** **A boat speeds towards the north at $6\sqrt{2}$ units. A man on the boat feels that the wind is blowing from the south-east at 5 units. Find the true velocity of the wind.**

$$\left[ \text{Ans : } \sqrt{157} \text{ units at an angle of } \pi - \cos^{-1}\left(\frac{5}{\sqrt{314}}\right) \text{ with east towards north} \right]$$

**( 43 )** **A steamer moves to the north-east with a speed of 40 units. A passenger on the steamer feels the wind to be blowing from the north with $25\sqrt{2}$ units. Find the true velocity of the wind.**

$$\left[ \text{Ans : } 5\sqrt{34} \text{ units at an angle of } \cos^{-1}\frac{4}{\sqrt{17}} \text{ with east towards south} \right]$$

**( 44 )** **A particle is displaced from A ( 2, 1 ) to B ( 4, 2 ) when forces of magnitudes $4\sqrt{5}$ in the direction $2\bar{i} + \bar{j}$ and $6\sqrt{5}$ in the direction $\bar{i} - 2\bar{j}$ are applied. Find the work done.**

**[ Ans: 20 units ]**