

(1) The end-points A and B of  $\overline{AB}$  are on the X- and Y-axis respectively. If  $AB = a + b$ ,  $a > 0$ ,  $b > 0$ ,  $a \neq b$  and P divides  $\overline{AB}$  from A in the ratio  $b : a$ , then show that P lies on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

(2) If the feet of the perpendiculars drawn to the tangent at any point of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  from foci S and S' are L and L' respectively, then show that  $SL \cdot S'L' = b^2$ .

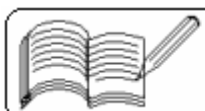
(3) Prove that the line segment of any tangent, between the tangents at the end-points of the major axis, forms a right angle at either focus of the ellipse.

(4) Show that the equation of the chord joining the points P ( $\alpha$ ) and Q ( $\beta$ ) of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $\frac{x}{a} \cos \frac{\alpha + \beta}{2} + \frac{y}{b} \sin \frac{\alpha + \beta}{2} = \cos \frac{\alpha - \beta}{2}$ .

(5) If the chord joining the points P ( $\alpha$ ) and Q ( $\beta$ ) of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  passes through the focus ( $ae, 0$ ), then prove that  $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{e - 1}{e + 1}$ .

(6) If the chord joining the points P ( $\alpha$ ) and Q ( $\beta$ ) of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  subtends a right angle at the centre, then show that  $\tan \alpha \cdot \tan \beta + \frac{a^2}{b^2} = 0$  and if it forms a right angle at the vertex ( $a, 0$ ), then show that  $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} + \frac{b^2}{a^2} = 0$ .

(7) If the difference of eccentric angles of the points P and Q on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $\frac{\pi}{2}$  and  $\overleftrightarrow{PQ}$  cuts intercepts of length c and d on the axes, then prove that  $\frac{a^2}{c^2} + \frac{b^2}{d^2} = 2$ .



(8) If two radii  $\overline{CP}$  and  $\overline{CQ}$  of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  are perpendicular, then prove that  $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} + \frac{1}{b^2}$ , where C is the centre of the ellipse.

(9) Find the equations of the tangents drawn to the ellipse,  $9x^2 + 16y^2 = 144$  from the point (2, 3).

[Ans:  $x + y - 5 = 0$ ,  $y - 3 = 0$ ]

(10) Find the equations of the tangents of the ellipse  $9x^2 + 4y^2 = 36$  parallel to the line  $y = 2x$ . Also obtain the co-ordinates of the contact points.

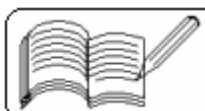
[Ans:  $2x - y + 5 = 0$  at  $\left(-\frac{8}{5}, \frac{9}{5}\right)$  and  $2x - y - 5 = 0$  at  $\left(\frac{8}{5}, -\frac{9}{5}\right)$ ]

(11) Show that the tangents at the end-points of a focal chord of the ellipse intersect on the directrix.

(12) Show that the point of intersection of the tangents to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at the points whose eccentric angles differ by  $\frac{\pi}{2}$  lies on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ .

(13) The difference of the eccentric angles of the points P and Q on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $\frac{\pi}{2}$ . If the tangents at P and Q intersect in R, then prove that  $\overline{CR}$  and  $\overline{PQ}$  bisect each other.

(14) P and Q are coherent points on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and its auxiliary circle. Show that the tangents to the ellipse at P and the circle at Q intersect on the X-axis. ( $a > b$ )



(15) If the lengths of the perpendicular line-segments from the centre to two mutually orthogonal tangents of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  are  $p_1$  and  $p_2$ , then prove that  $p_1^2 + p_2^2 = a^2 + b^2$ .

(16) A tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  intersects the axes in C and D respectively and touches the ellipse at mid-point of  $\overline{CD}$  in the first quadrant. Find its equation.

$$\left[ \text{Ans: } \frac{x}{a} + \frac{y}{b} = \sqrt{2} \right]$$

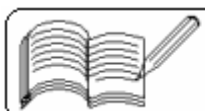
(17) If the perpendicular distance of the focus S from the tangent at a point P on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is p, then prove that  $SP = \frac{2ap^2}{b^2 + p^2}$ .

(18) B(0, b) is one end-point of the chord of an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ) passing through the focus S'. If P is another end-point, then show that the slope of  $\overline{CP}$  is  $\frac{(1 - e^2)^{\frac{3}{2}}}{2e}$ , where C is the centre of the ellipse.

(19) The foot of the perpendicular from a point P on ellipse to the major axis is M. If  $\overleftrightarrow{PM}$  intersects the tangent at the end-point of a latus rectum in R, then prove that  $MR = SP$ .

(20) If the line containing a focal-chord of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  intersects the auxiliary circle in Q and Q', then prove that  $SQ \times SQ' = b^2$ .

(21) Prove that if the tangent at a point P to the ellipse intersects a directrix at F, then  $\overline{PF}$  forms a right angle at the corresponding focus.



(22) The tangent at point P of an ellipse intersects the major axis in T. The line passing through T and perpendicular to major axis  $\overline{AA'}$ , intersects  $\overleftrightarrow{AP}$  and  $\overleftrightarrow{A'P}$  in Q and Q' respectively. Show that T is the mid-point of  $\overline{QQ'}$ .

(23) The tangent at point P of an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  intersects the axes in T and T' respectively. If R is the foot of perpendicular from the centre C to the tangent, then prove that  $TT' \cdot PR = a^2 - b^2$ .

(24) Find the condition that the line  $lx + my + n = 0$  may be a tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and find the co-ordinates of its point of contact.

$$\left[ \text{Ans: } a^2l^2 + b^2m^2 = n^2, \left( -\frac{a^2l}{n}, -\frac{b^2m}{n} \right) \right]$$

(25) If the tangent at any point of an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  with centre C meets the major axis in T and minor axis in T', then prove that  $\frac{a^2}{CT^2} + \frac{b^2}{CT'^2} = 1$  ( $a > b$ ).

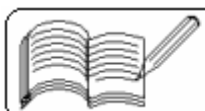
(26) Find the condition for the line  $x \cos \alpha + y \sin \alpha = p$  to be a tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

$$[\text{Ans: } p^2 \operatorname{cosec}^2 \alpha = a^2 \cot^2 \alpha + b^2]$$

(27) P and Q are corresponding points on an ellipse and its auxiliary circle respectively. If the tangent at P to the ellipse meets the major axis in T, then show that  $\overleftrightarrow{QT}$  is a tangent to the auxiliary circle.

(28) Find the perpendicular distance between the tangents to the ellipse  $\frac{x^2}{30} + \frac{y^2}{24} = 1$  which are parallel to the line  $4x - 2y + 23 = 0$ .

$$[\text{Ans: } 24 / \sqrt{5}]$$



(29) Prove that the equation of the chord of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  which has its mid-point at  $(h, k)$  is  $\frac{hx}{a^2} + \frac{ky}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$ .

(30) Prove that the equation of the chord joining the points  $P(\alpha + \beta)$  and  $Q(\alpha - \beta)$  on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $\frac{x}{a} \cos \alpha + \frac{y}{b} \sin \alpha = \cos \beta$ .

(31) Prove that the area of the triangle formed by the points  $P(\theta)$ ,  $Q(\alpha)$  and  $R(\beta)$  on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $2ab \left| \sin \frac{\alpha - \beta}{2} \sin \frac{\beta - \theta}{2} \sin \frac{\theta - \alpha}{2} \right|$ .

(32) Prove that the equations of the common tangents to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the circle  $x^2 + y^2 = r^2$  ( $b < r < a$ ) are  $y\sqrt{a^2 - r^2} = \pm x\sqrt{r^2 - b^2} \pm r\sqrt{a^2 - b^2}$ .

(33) Circles of constant radius  $c$  are drawn to pass through the ends of a variable diameter of the ellipse. Prove that the locus of their centres is the curve  $(x^2 + y^2)(a^2 x^2 + b^2 y^2 + a^2 b^2) = c^2(a^2 x^2 + b^2 y^2)$ .

(34) Prove that the measure of the angle between the two tangents drawn to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ from an external point } (h, k) \text{ is } \tan^{-1} \left| \frac{2ab \sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1}}{h^2 + k^2 - a^2 - b^2} \right|$$

