(1) The end-points A and B of $\overline{AB}$ are on the X- and Y-axis respectively. If AB = a + b, a > 0, b > 0, a ≠ b and P divides $\overline{AB}$ from A in the ratio b : a, then show that P lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(2) If the feet of the perpendiculars drawn to the tangent at any point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from foci S and S' are L and L' respectively, then show that $SL \cdot S'L' = b^2$.

(3) Prove that the line segment of any tangent, between the tangents at the end-points of the major axis, forms a right angle at either focus of the ellipse.

(4) Show that the equation of the chord joining the points P (α) and Q (β) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{x}{a}\cos\frac{\alpha + \beta}{2} + \frac{y}{b}\sin\frac{\alpha + \beta}{2} = \cos\frac{\alpha - \beta}{2}$.

(5) If the chord joining the points P (α) and Q (β) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the focus (ae, 0), then prove that $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{e - 1}{e + 1}$.

(6) If the chord joining the points P (α) and Q (β) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ subtends a right angle at the centre, then show that $\tan\alpha \cdot \tan\beta + \frac{a^2}{b^2} = 0$ and if it forms a right angle at the vertex (a, 0), then show that $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \frac{b^2}{a^2} = 0$.

(7) If the difference of eccentric angles of the points P and Q on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{\pi}{2}$ and $\overleftrightarrow{PQ}$ cuts intercepts of length c and d on the axes, then prove that $\frac{a^2}{c^2} + \frac{b^2}{d^2} = 2$.

(8) If two radii $\overline{CP}$ and $\overline{CQ}$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are perpendicular, then prove that $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} + \frac{1}{b^2}$, where C is the centre of the ellipse.

(9) Find the equations of the tangents drawn to the ellipse, $9x^2 + 16y^2 = 144$ from the point (2, 3).

[Ans: $x + y - 5 = 0$, $y - 3 = 0$]

(10) Find the equations of the tangents of the ellipse $9x^2 + 4y^2 = 36$ parallel to the line $y = 2x$. Also obtain the co-ordinates of the contact points.

$\left[\text{Ans: } 2x - y + 5 = 0 \text{ at } \left(-\frac{8}{5}, \frac{9}{5} \right) \text{ and } 2x - y - 5 = 0 \text{ at } \left(\frac{8}{5}, -\frac{9}{5} \right) \right]$

(11) Show that the tangents at the end-points of a focal chord of the ellipse intersect on the directrix.

(12) Show that the point of intersection of the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the points whose eccentric angles differ by $\frac{\pi}{2}$ lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

(13) The difference of the eccentric angles of the points P and Q on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{\pi}{2}$. If the tangents at P and Q intersect in R, then prove that $\overline{CR}$ and $\overline{PQ}$ bisect each other.

(14) P and Q are coherent points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its auxiliary circle. Show that the tangents to the ellipse at P and the circle at Q intersect on the X-axis. (a > b)

(15) If the lengths of the perpendicular line-segments from the centre to two mutually orthogonal tangents of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are p_1 and p_2, then prove that $p_1^2 + p_2^2 = a^2 + b^2$.

(16) A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersects the axes in C and D respectively and touches the ellipse at mid-point of $\overline{CD}$ in the first quadrant. Find its equation.

$\left[\text{Ans : } \frac{x}{a} + \frac{y}{b} = \sqrt{2} \right]$

(17) If the perpendicular distance of the focus S from the tangent at a point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is p, then prove that $SP = \frac{2ap^2}{b^2 + p^2}$.

(18) B (0, b) is one end-point of the chord of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b) passing through the focus S'. If P is another end-point, then show that the slope of $\overline{CP}$ is $\frac{\left(1 - e^2\right)^{\frac{3}{2}}}{2e}$, where C is the centre of the ellipse.

(19) The foot of the perpendicular from a point P on ellipse to the major axis is M. If $\overleftrightarrow{PM}$ intersects the tangent at the end-point of a latus rectum in R, then prove that MR = SP.

(20) If the line containing a focal-chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersects the auxiliary circle in Q and Q', then prove that $SQ \times SQ' = b^2$.

(21) Prove that if the tangent at a point P to the ellipse intersects a directrix at F, then $\overline{PF}$ forms a right angle at the corresponding focus.

(22) The tangent at point P of an ellipse intersects the major axis in T. The line passing through T and perpendicular to major axis $\overline{AA'}$, intersects $\overleftrightarrow{AP}$ and $\overleftrightarrow{A'P}$ in Q and Q' respectively. Show that T is the mid-point of $\overline{QQ'}$.

(23) The tangent at point P of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersects the axes in T and T' respectively. If R is the foot of perpendicular from the centre C to the tangent, then prove that $TT' \cdot PR = a^2 - b^2$.

(24) Find the condition that the line $lx + my + n = 0$ may be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and find the co-ordinates of its point of contact.

$\left[\text{Ans: } a^2l^2 + b^2m^2 = n^2, \quad \left(-\frac{a^2l}{n}, -\frac{b^2m}{n} \right) \right]$

(25) If the tangent at any point of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with centre C meets the major axis in T and minor axis in T', then prove that $\frac{a^2}{CT^2} + \frac{b^2}{CT'^2} = 1$ ($a > b$).

(26) Find the condition for the line $x \cos\alpha + y \sin\alpha = p$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

[Ans: $p^2 \operatorname{cosec}^2\alpha = a^2 \cot^2\alpha + b^2$]

(27) P and Q are corresponding points on an ellipse and its auxiliary circle respectively. If the tangent at P to the ellipse meets the major axis in T, then show that $\overleftrightarrow{QT}$ is a tangent to the auxiliary circle.

(28) Find the perpendicular distance between the tangents to the ellipse $\frac{x^2}{30} + \frac{y^2}{24} = 1$ which are parallel to the line $4x - 2y + 23 = 0$.

[Ans: $24 / \sqrt{5}$]

(29) Prove that the equation of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which has its mid-point at (h, k) is $\frac{hx}{a^2} + \frac{ky}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$.

(30) Prove that the equation of the chord joining the points $P(\alpha + \beta)$ and $Q(\alpha - \beta)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{x}{a}\cos\alpha + \frac{y}{b}\sin\alpha = \cos\beta$.

(31) Prove that the area of the triangle formed by the points $P(\theta)$, $Q(\alpha)$ and $R(\beta)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $2ab \left| \sin\frac{\alpha - \beta}{2} \sin\frac{\beta - \theta}{2} \sin\frac{\theta - \alpha}{2} \right|$.

(32) Prove that the equations of the common tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = r^2$ $(b < r < a)$ are $y\sqrt{a^2 - r^2} = \pm x\sqrt{r^2 - b^2} \pm r\sqrt{a^2 - b^2}$.

(33) Circles of constant radius c are drawn to pass through the ends of a variable diameter of the ellipse. Prove that the locus of their centres is he curve

$(x^2 + y^2)(a^2x^2 + b^2y^2 + a^2b^2) = c^2(a^2x^2 + b^2y^2)$.

(34) Prove that the measure of the angle between the two tangents drawn to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from an external point (h, k) is $\tan^{-1}\left| \frac{2ab\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1}}{h^2 + k^2 - a^2 - b^2} \right|$