(1) Show that $P(a, b+c)$, $Q(b, c+a)$ and $R(c, a+b)$ are collinear.

(2) Prove that the two lines joining the mid-points of the pairs of opposite sides and the line joining the mid-points of the diagonals of a quadrilateral are concurrent.

(3) If $(2, 3)$, $(4, 5)$ and $(a, 2)$ are the vertices of a right triangle, find a.

[Ans: 3, 7]

(4) Find the circumcentre of the triangle with vertices $(-1, 1)$, $(0, -4)$ and $(-1, -5)$ and deduce that the circumcentre of the triangle whose vertices are $(2, 3)$, $(3, -2)$ and $(2, -3)$ is the origin.

[Ans: $(-3, -2)$]

(5) For which value of a would the area of a triangle with vertices $(5, a)$, $(2, 5)$ and $(2, 3)$ be 3 units ?

[Ans: For any $a \in R$]

(6) Find the area of the triangle whose vertices are $(l^2, 2l)$, $(m^2, 2m)$ and $(n^2, 2n)$ if $l \neq m \neq n$.

[Ans: $|(l-m)(m-n)(n-l)|$]

(7) Find the area of the triangle whose vertices are $(5, 3)$, $(4, 5)$ and $(3, 1)$ and show that the triangle whose vertices are $(-2, 2)$, $(-3, 4)$ and $(-4, 0)$ has the same area.

[Ans: 3 units]

(8) Find the area of the triangle with vertices $(5, 3)$, $(4, 5)$ and $(3, 1)$ by shifting the origin at $(5, 3)$.

[Ans: 3 units]

(9) Prove that the mid-point of the segment joining the two points dividing $\overline{AB}$ from A in the ratios $m : n$ and $n : m$ is the mid-point of $\overline{AB}$.

(10) If P (1, 2) and Q (5, 6) divide $\overline{AB}$ from A in the ratios 2 : 1 and - 2 : 1, find the co-ordinates of A and B.

[Ans: A (- 1, 0), B (2, 3)]

(11) If (3, 2), (4, 5) and (2, 3) are three of the four vertices of a parallelogram, find the co-ordinates of the fourth vertex.

[Ans: (5, 4), (3, 6), (1, 0)]

(12) Show that the points (2, 3), (4, 5) and (3, 2) can be the vertices of a rectangle and find the co-ordinates of the fourth vertex.

[Ans: (5, 4)]

(13) If the mid-points of the sides of a triangle are (4, 3), (5, - 1) and (2, 7), find the vertices of the triangle.

[Ans: (7, - 5), (1, 11), (3, 3)]

(14) Find co-ordinates of the centroid, circumcentre and in-centre of the triangle whose vertices are (3, 4), (0, 4) and (3, 0).

$$\left[\text{Ans} : \left(2, \frac{8}{3} \right), \left(\frac{3}{2}, 2 \right), (2, 3) \right]$$

(15) A (3, 4), B (0, - 5) and C (3, - 1) are the vertices of triangle ABC. Determine the length of the altitude from A on $\overleftrightarrow{BC}$.

[Ans: 3 units]

(16) Points B (4, 1) and C (2, 5) are given. Find the equation of sets of all points P in the plane such that $m\angle BPC = \frac{\pi}{2}$. Find the set of all such points P.

[Ans: $\{ (x, y) \mid x^2 + y^2 - 6x + 6y + 13 = 0 \} - \{ B, C \}$]

(17) If $A(0, 1)$ and $B(2, 9)$ are given, find C on $\overleftrightarrow{AB}$ such that $AB = 3\ AC$.

$$\left[\text{Ans : } \left(-\frac{2}{3}, -\frac{5}{3} \right), \left(\frac{2}{3}, \frac{11}{3} \right) \right]$$

(18) Points $A(x_1, x_1 \tan\theta_1)$, $B(x_2, x_2 \tan\theta_2)$ and $C(x_3, x_3 \tan\theta_3)$ are given. If the circumcentre of triangle ABC is origin and its cenroid is (x, y), prove that

$$\frac{x}{y} = \frac{\cos\theta_1 + \cos\theta_2 + \cos\theta_3}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3} \quad \left(0 < \theta_1, \theta_2, \theta_3 < \frac{\pi}{2} \text{ and } x_1, x_2, x_3 > 0 \right)$$

(19) Prove that the mid-points of the sides of any quadrilateral are the vertices of a parallelogram.

(20) If D and G are respectively the mid-point of side $\overline{BC}$ and the centroid in triangle ABC, then prove that

(i) $AB^2 + AC^2 = 2\,(AD^2 + BD^2)$ and

(ii) $AB^2 + BC^2 + CA^2 = 3\,(GA^2 + GB^2 + GC^2)$

(21) Prove that the coordinates of all three vertices of an equilateral triangle cannot be rational numbers.

(22) If A, B, C and P are distinct non-collinear points of the plane, prove that Area of ΔPAB + Area of ΔPBC + Area of $\Delta PCA \geq$ Area of ΔABC

(23) If P is a point on the segment joining $A(3, 5)$ and $B(-5, 1)$ such that the area of ΔPOQ is 6 units where O is the origin and Q is the point $(-2, 4)$, then find the co-ordinates of P.

$$\left[\text{Ans : } (1, 4), \left(-\frac{19}{5}, \frac{8}{5} \right) \right]$$

(24) Prove that the area of the triangle formed by the mid-points of the sides of the triangle is one-fourth that of the original triangle.

(25) P (- 5, 1), Q (3, 5), B (1, 5) and C (7, - 2) are four points in the plane. The point A divides the segment $\overline{PQ}$ in the ratio $\lambda : 1$ from P. If the area of triangle ABC is 2 units, find the value of λ and also the co-ordinates of A.

$$\left[\text{Ans : } \lambda = 7,\ A\left(2, \frac{9}{2}\right); \quad \lambda = \frac{31}{9},\ A\left(\frac{6}{5}, \frac{41}{10}\right) \right]$$

(26) If the co-ordinates of A, B and P are (x_1, y_1), (x_2, y_2) and (x, y) respectively, and if A - P - B, then prove that $x + y$ lies between $x_1 + y_1$ and $x_2 + y_2$.

(27) Chord $\overline{CD}$ is parallel to the diameter $\overline{AB}$ of a given circle. P is any point on $\overline{AB}$. Prove that $PA^2 + PB^2 = PC^2 + PD^2$.

(28) If P is a variable point on the circumcircle of an equilateral triangle ABC, prove that the value of $AP^2 + BP^2 + CP^2$ is independent of the position of the point P.

(29) A straight line l intersects the lines $\overleftrightarrow{BC}$, $\overleftrightarrow{CA}$ and $\overleftrightarrow{AB}$ along the sides of triangle ABC respectively at P, Q and R. Prove that $\frac{BP}{PC} \cdot \frac{CQ}{QA} \cdot \frac{AR}{RB} = 1$.

(30) A variable rod of length l has one end A on X-axis and another end B on Y-axis. Prove that the equation of the set of points P which divide $\overline{AB}$ in the ratio 1 : 2 from A is $9x^2 + 36y^2 = 4l^2$.

(31) If A is $(a\cos\alpha, a\sin\alpha)$ and B is $(a\cos\beta, a\sin\beta)$, then find AB.

$$\left[\text{Ans : } 2\left| a \sin\left(\frac{\alpha - \beta}{2}\right)\right| \right]$$

(32) A (a, b) and B (c, d) are two points. If $\overline{AB}$ subtends an angle of measure θ at the origin, then prove that $\cos\theta = \dfrac{ac + bd}{\sqrt{(a^2 + b^2)(c^2 + d^2)}}$.

(33) If $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ and $S(a, 0)$ are three points, show that $\frac{1}{SP} + \frac{1}{SQ}$ is independent of t.

(34) For which value of k would the points $(k, 2 - 2k)$, $(-k + 1, 2k)$ and $(-4 - k, 6 - 2k)$ be distinct and collinear ?

[Ans: -1]

(35) A is $(-4, 0)$ and $B(4, 0)$. Find the locus of a point P such that the difference of its distances from A and B is 4.

[Ans: $3x^2 - y^2 = 12$]

(36) If the distance between the centroid and incentre of the triangle with vertices $(-36, 7)$, $(20, 7)$ and $(0, -8)$ is $\frac{25}{3}\sqrt{205}\,k$, then find the value of k.

$\left[\text{Ans : } k = \frac{1}{25}\right]$

(37) Prove that the locus of in-centre of a variable triangle whose one vertex is the origin and the other two vertices are on the co-ordinate axes is the set

$S = \{(x, y) \mid x^2 = y^2, \ x, y \in R\} - \{(0, 0)\}$.

(38) Prove that the locus of circumcentre of a variable triangle whose one vertex is the origin and the other two vertices are on the co-ordinate axes such that the side opposite to the origin is of constant length l is the set

$S = \{(x, y) \mid 4x^2 + 4y^2 = l^2\} - \left\{\left(\frac{l}{2}, 0\right), \left(-\frac{l}{2}, 0\right), \left(0, \frac{l}{2}\right), \left(0, -\frac{l}{2}\right)\right\}$.

(39) Prove that the locus of centroid of a variable triangle whose one vertex is the origin and the other two vertices are on the co-ordinate axes such that the side opposite to the origin is of constant length l is the set

$S = \{(x, y) \mid 9x^2 + 9y^2 = l^2\} - \left\{\left(\frac{l}{3}, 0\right), \left(-\frac{l}{3}, 0\right), \left(0, \frac{l}{3}\right), \left(0, -\frac{l}{3}\right)\right\}$.