

(1) Show that $P(a, b+c)$, $Q(b, c+a)$ and $R(c, a+b)$ are collinear.

(2) Prove that the two lines joining the mid-points of the pairs of opposite sides and the line joining the mid-points of the diagonals of a quadrilateral are concurrent.

(3) If $(2, 3)$, $(4, 5)$ and $(a, 2)$ are the vertices of a right triangle, find a .

[Ans: 3, 7]

(4) Find the circumcentre of the triangle with vertices $(-1, 1)$, $(0, -4)$ and $(-1, -5)$ and deduce that the circumcentre of the triangle whose vertices are $(2, 3)$, $(3, -2)$ and $(2, -3)$ is the origin.

[Ans: $(-3, -2)$]

(5) For which value of a would the area of a triangle with vertices $(5, a)$, $(2, 5)$ and $(2, 3)$ be 3 units?

[Ans: For any $a \in \mathbb{R}$]

(6) Find the area of the triangle whose vertices are $(l^2, 2l)$, $(m^2, 2m)$ and $(n^2, 2n)$ if $l \neq m \neq n$.

[Ans: $|(l - m)(m - n)(n - l)|$]

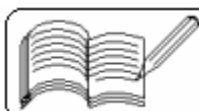
(7) Find the area of the triangle whose vertices are $(5, 3)$, $(4, 5)$ and $(3, 1)$ and show that the triangle whose vertices are $(-2, 2)$, $(-3, 4)$ and $(-4, 0)$ has the same area.

[Ans: 3 units]

(8) Find the area of the triangle with vertices $(5, 3)$, $(4, 5)$ and $(3, 1)$ by shifting the origin at $(5, 3)$.

[Ans: 3 units]

(9) Prove that the mid-point of the segment joining the two points dividing \overline{AB} from A in the ratios $m : n$ and $n : m$ is the mid-point of \overline{AB} .



(10) If $P(1, 2)$ and $Q(5, 6)$ divide \overline{AB} from A in the ratios $2 : 1$ and $-2 : 1$, find the co-ordinates of A and B .

[Ans: $A(-1, 0)$, $B(2, 3)$]

(11) If $(3, 2)$, $(4, 5)$ and $(2, 3)$ are three of the four vertices of a parallelogram, find the co-ordinates of the fourth vertex.

[Ans: $(5, 4)$, $(3, 6)$, $(1, 0)$]

(12) Show that the points $(2, 3)$, $(4, 5)$ and $(3, 2)$ can be the vertices of a rectangle and find the co-ordinates of the fourth vertex.

[Ans: $(5, 4)$]

(13) If the mid-points of the sides of a triangle are $(4, 3)$, $(5, -1)$ and $(2, 7)$, find the vertices of the triangle.

[Ans: $(7, -5)$, $(1, 11)$, $(3, 3)$]

(14) Find co-ordinates of the centroid, circumcentre and in-centre of the triangle whose vertices are $(3, 4)$, $(0, 4)$ and $(3, 0)$.

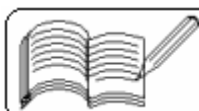
[Ans: $\left(2, \frac{8}{3}\right)$, $\left(\frac{3}{2}, 2\right)$, $(2, 3)$]

(15) $A(3, 4)$, $B(0, -5)$ and $C(3, -1)$ are the vertices of triangle ABC . Determine the length of the altitude from A on \overleftrightarrow{BC} .

[Ans: 3 units]

(16) Points $B(4, 1)$ and $C(2, 5)$ are given. Find the equation of sets of all points P in the plane such that $m\angle BPC = \frac{\pi}{2}$. Find the set of all such points P .

[Ans: $\{(x, y) \mid x^2 + y^2 - 6x + 6y + 13 = 0\} - \{B, C\}$]



(17) If $A(0, 1)$ and $B(2, 9)$ are given, find C on \overleftrightarrow{AB} such that $AB = 3 AC$.

$$\left[\text{Ans: } \left(-\frac{2}{3}, -\frac{5}{3} \right), \left(\frac{2}{3}, \frac{11}{3} \right) \right]$$

(18) Points $A(x_1, x_1 \tan \theta_1)$, $B(x_2, x_2 \tan \theta_2)$ and $C(x_3, x_3 \tan \theta_3)$ are given. If the circumcentre of triangle ABC is origin and its centroid is (x, y) , prove that

$$\frac{x}{y} = \frac{\cos \theta_1 + \cos \theta_2 + \cos \theta_3}{\sin \theta_1 + \sin \theta_2 + \sin \theta_3} \quad (0 < \theta_1, \theta_2, \theta_3 < \frac{\pi}{2} \text{ and } x_1, x_2, x_3 > 0)$$

(19) Prove that the mid-points of the sides of any quadrilateral are the vertices of a parallelogram.

(20) If D and G are respectively the mid-point of side \overline{BC} and the centroid in triangle ABC , then prove that

$$\begin{aligned} \text{(i)} \quad & AB^2 + AC^2 = 2(AD^2 + BD^2) \quad \text{and} \\ \text{(ii)} \quad & AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2) \end{aligned}$$

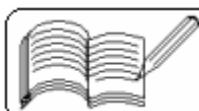
(21) Prove that the coordinates of all three vertices of an equilateral triangle cannot be rational numbers.

(22) If A, B, C and P are distinct non-collinear points of the plane, prove that Area of $\triangle PAB$ + Area of $\triangle PBC$ + Area of $\triangle PCA \geq$ Area of $\triangle ABC$

(23) If P is a point on the segment joining $A(3, 5)$ and $B(-5, 1)$ such that the area of $\triangle POQ$ is 6 units where O is the origin and Q is the point $(-2, 4)$, then find the coordinates of P .

$$\left[\text{Ans: } (1, 4), \left(-\frac{19}{5}, \frac{8}{5} \right) \right]$$

(24) Prove that the area of the triangle formed by the mid-points of the sides of the triangle is one-fourth that of the original triangle.



- (25) P (-5, 1), Q (3, 5), B (1, 5) and C (7, -2) are four points in the plane. The point A divides the segment \overline{PQ} in the ratio $\lambda : 1$ from P. If the area of triangle ABC is 2 units, find the value of λ and also the co-ordinates of A.

$$\left[\text{Ans: } \lambda=7, A\left(2, \frac{9}{2}\right); \lambda = \frac{31}{9}, A\left(\frac{6}{5}, \frac{41}{10}\right) \right]$$

- (26) If the co-ordinates of A, B and P are (x_1, y_1) , (x_2, y_2) and (x, y) respectively, and if A - P - B, then prove that $x + y$ lies between $x_1 + y_1$ and $x_2 + y_2$.

- (27) Chord \overline{CD} is parallel to the diameter \overline{AB} of a given circle. P is any point on \overline{AB} . Prove that $PA^2 + PB^2 = PC^2 + PD^2$.

- (28) If P is a variable point on the circumcircle of an equilateral triangle ABC, prove that the value of $AP^2 + BP^2 + CP^2$ is independent of the position of the point P.

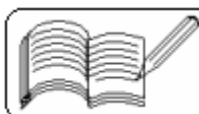
- (29) A straight line l intersects the lines \overleftrightarrow{BC} , \overleftrightarrow{CA} and \overleftrightarrow{AB} along the sides of triangle ABC respectively at P, Q and R. Prove that $\frac{BP}{PC} \cdot \frac{CQ}{QA} \cdot \frac{AR}{RB} = 1$.

- (30) A variable rod of length l has one end A on X-axis and another end B on Y-axis. Prove that the equation of the set of points P which divide \overline{AB} in the ratio 1 : 2 from A is $9x^2 + 36y^2 = 4l^2$.

- (31) If A is $(a \cos \alpha, a \sin \alpha)$ and B is $(a \cos \beta, a \sin \beta)$, then find AB.

$$\left[\text{Ans: } 2 \left| a \sin\left(\frac{\alpha - \beta}{2}\right) \right| \right]$$

- (32) A (a, b) and B (c, d) are two points. If \overline{AB} subtends an angle of measure θ at the origin, then prove that $\cos \theta = \frac{ac + bd}{\sqrt{(a^2 + b^2)(c^2 + d^2)}}$.



(33) If $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ and $S(a, 0)$ are three points, show that $\frac{1}{SP} + \frac{1}{SQ}$ is independent of t .

(34) For which value of k would the points $(k, 2 - 2k)$, $(-k + 1, 2k)$ and $(-4 - k, 6 - 2k)$ be distinct and collinear?

[Ans: - 1]

(35) A is $(-4, 0)$ and B $(4, 0)$. Find the locus of a point P such that the difference of its distances from A and B is 4.

[Ans: $3x^2 - y^2 = 12$]

(36) If the distance between the centroid and incentre of the triangle with vertices $(-36, 7)$, $(20, 7)$ and $(0, -8)$ is $\frac{25}{3}\sqrt{205}k$, then find the value of k .

[Ans: $k = \frac{1}{25}$]

(37) Prove that the locus of in-centre of a variable triangle whose one vertex is the origin and the other two vertices are on the co-ordinate axes is the set

$$S = \{(x, y) \mid x^2 = y^2, x, y \in \mathbb{R}\} - \{(0, 0)\}.$$

(38) Prove that the locus of circumcentre of a variable triangle whose one vertex is the origin and the other two vertices are on the co-ordinate axes such that the side opposite to the origin is of constant length l is the set

$$S = \{(x, y) \mid 4x^2 + 4y^2 = l^2\} - \left\{\left(\frac{l}{2}, 0\right), \left(-\frac{l}{2}, 0\right), \left(0, \frac{l}{2}\right), \left(0, -\frac{l}{2}\right)\right\}.$$

(39) Prove that the locus of centroid of a variable triangle whose one vertex is the origin and the other two vertices are on the co-ordinate axes such that the side opposite to the origin is of constant length l is the set

$$S = \{(x, y) \mid 9x^2 + 9y^2 = l^2\} - \left\{\left(\frac{l}{3}, 0\right), \left(-\frac{l}{3}, 0\right), \left(0, \frac{l}{3}\right), \left(0, -\frac{l}{3}\right)\right\}.$$

